UNIV. OF MICH.

NUMBER 6

# THE PHYSICAL REVIEW

*A JOURNAL OF EXPERIMENTAL AND THEORETICAL PHYSICS*

CONDUCTED BY

THE AMERICAN PHYSICAL SOCIETY

BOARD OF EDITORS

FREDERICK BEDELL, Managing Editor

| | | |
|---|---|---|
| J. S. AMES | E. P. LEWIS | N. E. DORSEY |
| E. BUCKINGHAM | W. C. SABINE | WM. DUANE |
| A. A. MICHELSON | A. TROWBRIDGE | O. M. STEWART |

## MAY, 1917

On the So-called Magnetic Rays of Right; Electrical Oscillations in Geissler Tubes; and the Periodic Interruption of Spark Discharges. JAMES E. IVES 349

The Single-Lined and the Many-Lined Spectrum of Mercury. T. C. HEBB 371

Theoretical Considerations relating to the Single-Lined and the Many-Lined Spectrum of Mercury. R. A. MILLIKAN 378

On the Nuclear Charges of Atoms. FERNANDO SANFORD 383

The Magnetic Properties of Iron, Nickel and Cobalt above the Curie Point, and Kesson's Quantum Theory of Magnetism. EARLE M. TERRY 394

The Measurement of Magnetic Permeability. ARTHUR WHITMORE SMITH 414

Time Lag in Magnetization. ARTHUR WHITMORE SMITH 419

A Study of the Accelerated Motion of Small Drops through a Viscous Medium. W. A. SHEWHART 425

The Use of the Einthoven Galvanometer and a Detector of the Audion Type for Measuring the Strength of Radiotelegraphic Signals. LAURENS E. WHITTEMORE 434

Proceedings of the American Physical Society 437

A Simple Method for Determining the Audibility Current of a Telephone Receiver. *Edward W. Washburn*

NEW BOOKS 440

PUBLISHED BY

THE AMERICAN PHYSICAL SOCIETY

LANCASTER, PA., AND ITHACA, N. Y.

Entered at the post-office at Lancaster, Pa., as second class matter

A. B. COLE
Secretary J. B. AMES
E. MERRILL
Local Secretary for the Pacific Coast E. P. LEWIS University of California, Berkeley, Cal.

The Council of the Society consists of the president, vice-president, secretary, treasurer, all living Past Presidents and eight elected members as follows:

Past Presidents: A. A. Michelson, A. G. Webster, Carl Barus, E. L. Nichols, Henry Crew, W. F. Magie, Ernest Merritt.

Elected Members: G. K. Burgess, D. C. Miller; C. E. Mendenhall, C. W. Stewart; Irving Langmuir, G. B. Pegram; G. O. Squier, H. A. Wilson.

# THE PHYSICAL REVIEW

Annual Subscription, throughout the Postal Union, $6.00
Single Copies, postpaid, $ .60

THE PHYSICAL REVIEW is conducted by a BOARD OF EDITORS, named on the outside cover.

Subscriptions should be sent to THE PHYSICAL REVIEW, *or North Queen Street, Lancaster, Pa., or Ithaca, N. Y.*

Two volumes of THE PHYSICAL REVIEW are published annually, these volumes beginning the January and July, respectively, and containing six numbers each.

A limited number of complete sets of the REVIEW, both First and Second Series, will be supplied at the price of $3.00 per volume for all volumes, postage extra. When they can be furnished without breaking a set, separate back volumes will be supplied at the same price, and single back numbers will be mailed to any address in the Postal Union at the price of $.60 per copy.

Back numbers will be furnished to members at a discount of ten per cent., provided a statement of membership accompanies the order. When a member's name has been dropped from the mailing list of the Review on account of non-payment of dues, his name will be restored to the mailing list when dues are paid; and any numbers that he has failed to receive by that time will be furnished without cost upon application to the Review.

*First Series.* From July 1, 1893, to January 1, 1913, the REVIEW was conducted by Edward L. Nichols, Ernest Merritt and Frederick Bedell, with the financial support of Cornell University. During this period thirty-five volumes were issued, constituting the First Series. The first four volumes of the First Series were issued monthly; subsequent volumes were issued bimonthly.

*Second Series.* On January first, 1913, the REVIEW was transferred to the American Physical Society. A second series was then begun, the January, 1913, number being designated Volume I, Number 1, Second Series. (An announcement of this number will be found on the first page of the new series.)

For advertising rates, address THE PHYSICAL REVIEW, Ithaca, N. Y. Change of copy for standing advertisements should be sent direct to the printers, THE NEW ERA PRINTING CO., Lancaster, Pa.

Contributors to the REVIEW are referred to page xiv of the advertisements for further information.

Permission is given to republish any article appearing in the REVIEW, provided proper credit is given.

Address Correspondence to

THE PHYSICAL REVIEW, ITHACA, N. Y.

*Second Series.* *May, 1917.* *Vol. IX., No. 5*

THE

# PHYSICAL REVIEW.

## ON THE SO-CALLED MAGNETIC RAYS OF RIGHI; ELECTRICAL OSCILLATIONS IN GEISSLER TUBES, AND THE PERIODIC INTERRUPTION OF SPARK DISCHARGES.[1]

By James E. Ives.

### 1. Introduction.

In 1908, Righi[2] described some experiments upon which he based the assumption of the existence of a new kind of ray in a Geissler tube acted upon by a magnetic field. Each ray was supposed to be made up of an electron revolving around a positive ion, each positive ion with its accompanying electron forming as it were a double star. The rays were supposed to be formed under the action of a magnetic field, and under its action to give rise to a number of unusual phenomena in the tube.

The object of the experiments described below was, if possible, to

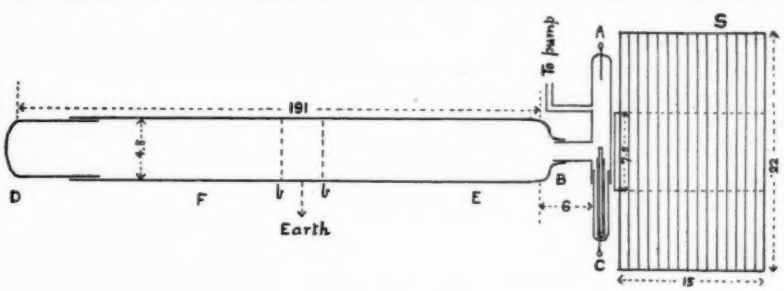



Fig. 1.

obtain an explanation of these phenomena in terms of well-known principles without having to resort to the assumption of a new kind of ray.

The tube which I used in most of my experiments is shown diagram-

[1] For an abstract of part of this paper, see Physical Review, 7, 407–409, 1916.

[2] Rendiconti della R. Accad. dei Lincei, 2 Feb., 1908; Mem. della R. Accad. della Sci. Bologna, 17, 87–90, 1908; La Materia Radiente e i Raggi Magnetici, Bologna, 1909; Strahlende Materie und Magnetische Strahlen, Leipsig, 1909.

matically in Figs. 1 and 2; Fig. 2 is an enlarged drawing of a portion of Fig. 1. The dimensions are given on the figures in centimeters. The tube, in a general way, is similar to the one used by Righi, but is somewhat simpler in design. It consists of a small tube, Fig. 2, having the anode at *A* and the cathode at *C*. Opening into this, at *B*, is a much longer and wider tube, *BD*, Fig. 1. The tube *AC* I will call the *side tube*, and the tube *BD* the *long tube*. The anode is a platinum wire .046 cm. in diameter; and the cathode is an aluminum wire, .328 cm. in diameter, covered to within a centimeter of its tip with a small glass tube to confine the discharge to its end.

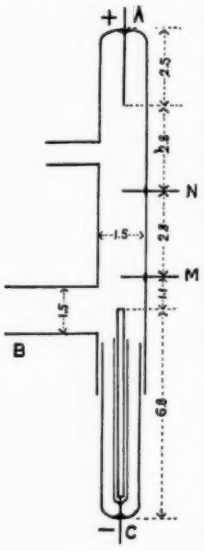


Fig. 2.

The whole tube, which we may call a Righi tube, is connected in series with a high potential battery, *B*, and a non-inductive resistance of from half a million to a million ohms, *R*, as shown in Fig. 3. *kk* is a double switch which connects the battery to the rest of the circuit.

A static induction machine can be used in place of the high-potential battery, but the battery is better as it gives a steadier and larger current. The battery had 2,000 cells, and I usually used about 2,100 or about 2,700 volts.

In all the experiments, I used the air of the room and did not attempt to dry it. To exhaust the tube I used a Gaede rotating mercury pump, or a Gaede oil pump; and to measure the pressure, a MacLeod gauge. The Righi effects were best obtained at pressures varying from .1 to .2 mm. of mercury.

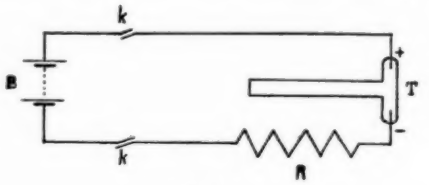


Fig. 3.

To produce the magnetic field, I used a solenoid *S*, Fig. 1, having a soft iron core, which gave an approximately uniform transverse field in the region between the cathode and anode of the tube. Its strength could be varied from zero to 2,600 gausses.

When no magnetic field is present, there is the usual Geissler tube appearance in the side tube *AC*, that is a striated pink glow in the neighborhood of the anode and a bluish-violet glow in the neighborhood of the cathode, but there is no glow anywhere in the long tube. As the strength of the magnetic field is gradually increased, the pink striæ are pressed more and more against the side of the tube, their

number increases, and they move downwards until they fill the whole of the Faraday dark space. When the field reaches a certain strength, depending upon the pressure of the air in the tube, the striæ coalesce, forming a continuous pink column, and the discharge through the tube becomes periodic, emitting a note. At the same time, there appears in the long tube, *BD*, a bluish-violet glow extending from the cathode to *E*, see Fig. 1; a pinkish glow extending from *E* to *F*, and a bluish-white glow extending from *F* to *D*. The short bluish-violet column is either attracted to, or repelled from, the north pole of an auxiliary magnet, and produces a spot of fluorescence at the point where it touches the tube. This bluish-violet column Righi assumes to be made up of his magnetic rays. The pinkish column is bent upwards in an arc by the north pole of an auxiliary magnet, and the bluish-white column is bent downwards in an arc. The directions in which these columns are bent seem to show that a current flows away from the region of *F*, on both sides of it. Righi therefore calls this region *F* the "virtual anode." The column from *E* to *D*, we may call the "Righi column." Righi assumes that it is produced by the disintegration of the magnetic rays. He assumes that the magnetic rays under the action of the magnetic field move out along the lines of force. When they have reached the weaker parts of the field they are supposed to disintegrate, producing free positive and negative ions. The positive ions so produced are assumed to form the virtual anode. The portion of the Righi column from *E* to *F*, I shall call the "positive column" because it resembles in color the positive column in the ordinary Geissler tube, and the portion from *F* to *D*, I shall call, by contrast, the "negative column."

In the region of the virtual anode, for several centimeters, the column is often double, there being both a positive and negative column present at the same time. In this case, it presents much the appearance of a vibrating string. Occasionally the column will be double throughout the whole length of the tube: for instance, on one occasion the negative column was of about equal brightness throughout the whole length of tube, and the positive column decreased in brightness from the fixed to the free end of the tube, being very faint towards the free end.

The virtual anode may exist at any point, *F*, along the tube. As the strength of the magnetic field increases, it moves out along the tube toward *D*. Under certain conditions, as for example when an electrostatic induction machine is used, *F* may be close to *E*, and the negative column may occupy nearly the whole length of the tube.

The "magnetic rays" have been investigated by Thirkell,[1] More and

[1] Proc. Roy. Soc., 83A, 324–334, 1910.

Rieman,[1] More and Mauchly[2] and others. Both Thirkell and More and Mauchly have come to the conclusion that they are negatively charged particles moving in spirals along the lines of magnetic force. In fact More and Mauchly state that they have found that all of the Righi effects can be obtained without a magnetic field by using a Wehnelt hot lime cathode.[3] Recently the Righi effects have been discussed by Holm[4] and Righi.[5]

The bluish-violet column from *B* to *E*, in agreement with the investigations of Thirkell and More and Mauchly, we may assume to consist of cathode rays, since it begins at the cathode and makes the glass fluorescent where it touches it. These investigations do not explain, however, the existence of the positive and negative columns, the virtual anode, and the periodicity of the discharge.

I first repeated the experiments described by Righi in his publications, and my experiments confirmed his experimental results. Thus, I found that the length of the Righi column depended upon the strength of the magnetic field, increasing as the strength of the field increased; and that the frequency of interruption of the discharge decreased as the strength of the field increased. The frequency could in this way be varied through several octaves.

## 2. Capacity of the Long Tube.

Early in my experiments, I noticed that when the finger was touched to a metal band holding the long tube in place the note emitted by the tube became very much louder and the frequency of the discharge was lowered, the frequency normally being very high, about 2,000 a second.

I therefore wrapped a brass band, 10 cm. wide, *bb*, Fig. 1, around the tube and connected it to earth. The frequency was then reduced to about one fourth of its original value. When this band was made twice as wide, the frequency was again reduced. By decreasing the width of the band, and also the strength of the magnetic field, the pitch can be made to vary from zero, or say one discharge every ten seconds, to a pitch above the limits of audibility. In other words, with an earthed metal band on the tube, any frequency that is desired can be obtained.

[1] Phil. Mag., 24, 307–316, 1912.

[2] Phil. Mag., 26, 252–267, 1913.

[3] R. Reiger, Sitzungsberichte der Physikalisch-medizinischen Sozietät in Erlangen, 37, 20, 21, 1905, states that when using a Wehnelt cathode it is very difficult to get a continuous discharge. This fact, as will be shown later, is probably the reason why More and Mauchly obtained the Righi effects with this kind of a cathode.

[4] Phys. Zeitschr., 16, 79, 80, 1915.

[5] Nuovo Cimento, 9, Gennaio—Febbraio, 1916.

When the band was on the tube, there was a negative column from *bb* to *D*, and, usually, a *double* column from *bb* to *E*. By a "double" column, I mean that the auxiliary magnet divides the luminous column into two parts, one part being bent up and the other down.

These experiments show that the frequency of the discharge depends not only upon the strength of the magnetic field, but also upon the capacity of the long tube. If we assume that the long tube acts as a conducting cylinder, its calculated capacity would be about 20 cm. in electrostatic units. Other experiments, which will be described later, would seem to indicate that this is of the order of the maximum effective capacity of the tube.[1]

The connection between the strength of the magnetic field and the frequency of the discharge probably depends upon the fact that the length and strength of the stream of electrons proceeding from the cathode, into the long tube, increases with the strength of the field, making the effective capacity of the tube greater, that part of the tube only being effective as capacity where the gas has become ionized by the presence of the electrons.

### 3. ACTION OF THE MAGNETIC FIELD.

The component of the magnetic field transverse to the side tube, due to the solenoid *S*, Fig. 1, was approximately uniform between the electrodes; not varying more than 10 per cent. along the line joining them. It was transverse to the line joining the electrodes, and in the direction of the axis of the long tube. Its action is twofold: (1) it produces a periodic interruption of the discharge in the side tube, and (2) it sends a periodic charge of electrons into the long tube.

The first fact is shown by the following experiment. I disconnected the long tube from the side tube at *B*, Fig. 1, and closed the aperture so formed with a glass plug. I then examined the action of the magnetic field upon the discharge in the side tube. Using a rotating mirror to observe it, I found that for a certain strength of the field the discharge through the tube became periodic, the frequency decreasing as the strength of the field increased, and varying through several octaves. When the field reached a certain strength, the discharge in the tube ceased. The *periodicity* of the Rhigi effect is, therefore, due to the action of the field upon the discharge through the *side tube*. Its frequency is modified by the presence of the long tube, *BD*, but the periodicity does not depend for its existence upon it.

[1] For discussions of the capacity of a Geissler tube, see: G. Wiedermann, Die Lehre von der Elektricitat, Bd. 4, 505–511, 1885; J. Borgmann, Phys. Zeits., 2, 651–653, 1901; S. N. Taylor, PHYS. REV., 18, 338–344, 1904; R. Reiger, l.c., 21–39.

The periodic discharge of electrons into the long tube is shown by the existence of the cathode stream, already described, from the cathode to $E$.

Repeated experiments showed that the Righi effects are never present in the long tube unless the discharge through the side tube is an interrupted one.

It was also found that even if the discharge is an interrupted one, the Righi effects are not produced unless the cathode is opposite the opening in the long tube at $B$, Fig. 1. If the distance between the electrodes is kept constant, but the cathode is lowered until it is 3 or 4 centimeters below the opening at $B$, the effects are not obtained. This would seem to show that there must be a projection of the electrons directly from the *cathode* into the long tube. The action of the same field upon the *positive column*, when a portion of it is opposite the opening at $B$, does not produce the same effect.

Table I. shows how the potential difference, between the anode and the

TABLE I.

| Pressure in Mm. of Mercury. | Field in Gausses. | Potential Across Tube in Volts. | | Difference. | Current in Milliamperes. | | Difference. | |
|---|---|---|---|---|---|---|---|---|
| | | Without Field. | With Field. | | Without Field. | With Field. | | |
| .32 | 200 | 750 | 650 | −100 | 1.00 | 1.09 | +.09 | Long tube |
| | 245 | 780 | 680 | −100 | 1.05 | 1.15 | +.10 | disconnected |
| | 265 | 800 | 700 | −100 | 1.02 | 1.15 | +.13 | from side |
| | 290 | Discharge became discontinuous. | | | | | | tube. |
| .20 | 265 | 1,050 | 800 | −250 | .75 | .95 | +.20 | |
| | 300 | Discharge became discontinuous. | | | | | | |
| .15 | 340 | 900 | 750 | −150 | 1.10 | 1.20 | +.10 | Long tube |
| | 410 | Discharge became discontinuous. | | | | | | connected |
| .13 | 170 | 1,040 | 850 | −190 | .84 | 1.00 | +.16 | to side |
| | 220 | 1,010 | 790 | −220 | .78 | .98 | +.20 | tube. |
| | 280 | Discharge became discontinuous. | | | | | | |

cathode, and the current through the tube vary as the magnetic field is increased. Two cases are given, first, when the long tube, $BD$, was *disconnected* from the side tube at $B$, and, second, when the long tube was *not disconnected*. The differences of potential were measured with a quadrant electrometer, and the currents with a milliammeter.

To determine the distribution of the potential in the side tube, and the effect of a magnetic field upon it, platinum wires were sealed into the walls of the side tube at the points $M$ and $N$, Fig. 2. The wire $M$ was in the negative glow, just outside the Crookes dark space, and the wire $N$ was just outside the positive striations. The difference of potential

between $M$ and the cathode was approximately equal to the cathode fall. The results, given in Table II., were obtained when the long tube was *disconnected* from the side tube.

It is seen from the two tables that *as long as the discharge through the side tube is continuous*, the effect of the transverse magnetic field is to

TABLE II.

*The Long Tube Disconnected from the Side-Tube.*

| Pressure in Mm. of Mercury. | Field in Gausses. | Potential Difference Between | Potential Across Tube in Volts. | | Difference. | Current in Milliamperes. | | Difference. |
|---|---|---|---|---|---|---|---|---|
| | | | Without Field. | With Field. | | Without Field. | With Field. | |
| .20 | 135 | $C$ and $A$ | 1,080 | 1,020 | −60 | .80 | .84 | +.04 |
| | | $C$ and $N$ | 940 | 890 | −50 | .76 | .79 | +.03 |
| | | $C$ and $M$ | 950 | 860 | −90 | .78 | .80 | +.02 |

considerably *decrease* the difference of potential between the electrodes of the tube, and to correspondingly *increase* the current flowing through it. It is also clearly seen that when the field reaches a certain strength the discharge becomes discontinuous. Readings of the electrometer and milliammeter for field strengths greater than this obviously have no value. Table II. also shows that the cathode fall of potential amounts to about nine tenths of the total fall of potential between the electrodes. Just why the magnetic field for a certain strength should extinguish the discharge, it is difficult to say.[1] The fact that the potential decreases as the magnetic field increases until the discharge becomes interrupted, suggests that the action of the field is to reduce the potential until it becomes less than the "maintenance potential," *i. e.*, the minimum potential necessary to *maintain* the current through the tube. The value of the maintenance potential will depend upon the shape of the tube and its electrodes, the pressure of the air in it, the value of the impressed electromotive force and resistance in the circuit, and probably also upon the strength of the magnetic field.

The *minimum potential* curve, $A$, and the *maintenance potential* curve, $B$, for varying pressure were obtained for the tube and are shown in Fig. 4. The minimum potential curve, $A$, shows the smallest potential that will *start* a discharge through the tube. Curve $B$ was obtained by gradually decreasing the potential applied to the circuit, by a potentiometer arrangement, until the discharge through the tube stopped.

[1] See Thomson, Conduction of Electricity through Gases, 2d edition, pages 576–579; Riecke, Ann. Phys., 4, 592–616, 1901; Stark, Ann. Phys., 12, 31–51, 1903; Schwienhorst, Diss. Gottingen, 1903; Earhart, PHYS. REV., 3, 103–114, 1914.

There was a non-inductive resistance in the circuit of about half a million ohms, and the current flowing through it varied from .34 to .13 of a milliampere.

### 4. Sign of the Charges Inside the Long Tube.

The charges on the *inside* of the long tube were examined both with a brass band touching the walls of the tube, and with a platinum wire placed in its axis. The brass band was 5 cm. wide and was attached to a wire sealed into the end of a long glass tube 8 mm. in diameter which could be introduced into a suitable holder at the free end *D* of the long tube. In this way it could be placed at any desired place in the tube. The platinum wire, .046 cm. in diameter and 5.5 cm. in length, was

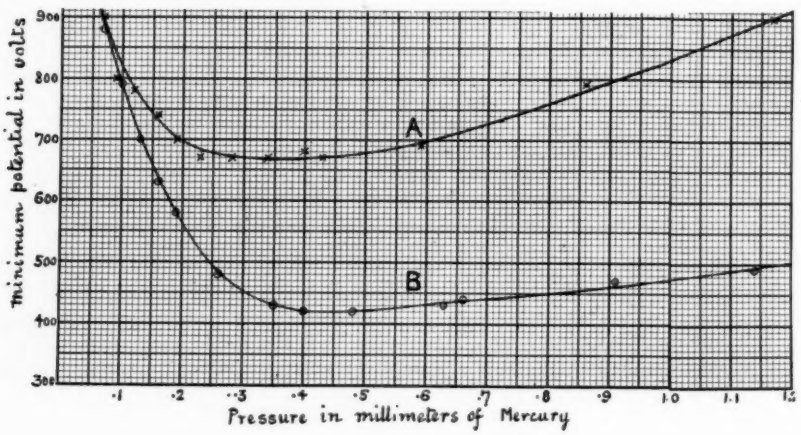



Fig. 4.

similarly sealed into the end of a long glass tube. The band, or wire, was connected to an aluminum leaf electroscope, and the sign of the charge on it was determined by determining the sign of the charge on the electroscope. The results obtained were the same both for the band and the wire, and were briefly the following.

If the magnetic field was not strong enough to make the Righi column long enough to touch the brass band, or wire, the behavior of the electroscope was irregular, but if it was strong enough, on closing the high potential key, *kk*, Fig. 3, with no magnetic field, I got a *positive* charge on the electroscope; then with the magnetic field on I got a deflection of *indeterminate* sign, that is a deflection due to a charge rapidly alternating in sign; and finally opening the high potential key, still keeping the magnetic field on, I got a residual *negative* charge. So then, I had an initial positive charge, then a charge rapidly alternating in sign, and finally a residual negative charge. The farther the band, or wire, is from

the side tube the more slowly does the leaf charge up for the initial positive charge. The behavior of the electroscope, therefore, indicates that when the magnetic field is strong enough to make the discharge interrupted there is a charging of the tube alternately positively and negatively, and that it probably begins with a positive and ends with a negative charge.

The residual negative charge may be the result of an oscillation due to the capacity of the long tube, or it may be due to the magnetic field diverting a stream of electrons into it. The initial positive charge is explained by the fact that the long tube always has a positive charge, of a greater or less value, when there is a steady discharge through the tube. Since nine tenths of the total fall of potential across the electrodes takes place in the neighborhood of the cathode, the whole of the long tube must be very nearly at the potential of the anode. In order to have this potential it must take on a positive charge. Therefore, when the discharge passes through the side tube, the long tube will begin to take on a positive charge. It will continue to do this until the discharge through the side tube becomes interrupted, when its positive charge will flow back towards the cathode, and may or may not produce a complete electrical oscillation in the long tube. That high frequency oscillations are possible in Geissler tubes is shown by an independent experiment which will be described later in this paper.

If an earthed brass band is placed around the outside of the long tube, causing the separate interruptions to succeed each other very slowly, for instance one every four seconds, the discharge builds up gradually, the glow first appearing faintly in the long tube from *bb* to *B*, Fig. 1, growing brighter and brighter and then suddenly flashing up in the side tube. When this flash takes place, the deflection of the electroscope takes place. Immediately after the flash, all the glow both in the long tube and the side tube goes out, and the action repeats itself.

As seen in a rotating mirror, both the positive and negative parts of the Righi column and the discharge in the side tube appear to be synchronous. If the two parts of the Righi column are not perfectly synchronous, and I do not think that they are, the interval of time between their separate occurrences must be very much less than the audible period of the discharge in the tube, probably less than .0001 second.

It is significant that, even when the magnetic field is not on, every time the high potential key, *kk*, is either closed or opened there is an instantaneous flash of light in the long tube.

5. THE PRODUCTION OF THE RIGHI EFFECTS WITH A SPARK GAP.

While experimenting with the arrangement of the circuit shown in Fig. 3, I discovered, accidentally, that all the Righi effects could be obtained without any magnetic field if a suitable spark gap was placed in the circuit. The discovery resulted from the fact that there was a loose contact, and consequently sparking, at one contact of the double switch *kk*. I noticed that when the sparking occurred there was luminosity in the long tube which much resembled the Righi effect. I did not, at first, pay much attention to it, as Righi in his book, and in subsequent papers,[1] has spoken of the importance of not having a loose contact in the circuit, as this would, according to him, produce a spurious effect. I thought, first of all, that I had a spurious effect, but, by putting into the circuit a suitable spark gap on the anode side, and a high resistance on the cathode side of the tube, I found that I could get all the Righi effects without any magnetic field.

At first, not all spark gaps would give the effect. The terminals of the gap must, apparently, be of relatively large area, and be so arranged that the length of the gap remains approximately constant. It is probably necessary that the gap shall be so constructed that the terminals cool off rapidly, preventing arcing. Later I had no difficulty in obtaining the effect with terminals of large surface constructed of zinc, copper or brass. The terminals which I finally adopted were zinc cylinders with flat ends, 9.3 mm. long and 12 mm. in diameter. They were not used end on, that is with their flat faces parallel to each other, but with the flat ends making an angle of 45° with each other, so that the sparking took place between two adjacent edges, as this arrangement seemed to give more regular results.

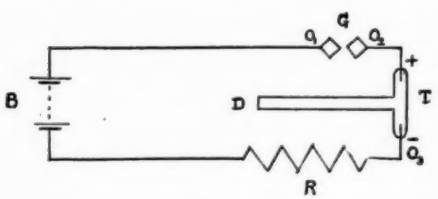



Fig. 5.

The circuit used is shown diagrammatically in Fig. 5. It will be noted that there are four elements in it, the battery *B*, the gap *G*, the tube *T*, and the high resistance *R*. In order that well-developed and characteristic Righi effects shall be produced, I found that the gap must be placed next to the anode of the tube, and the high resistance next to the cathode of the tube. No other arrangement will give the effects completely. For instance, if *G* and *R* are interchanged, the characteristic Righi effects are not obtained, nor are they obtained if *G* and *R* are both put on the same side of the tube. On the other hand, if the

[1] For this and other interesting matters, see Righi, Phys. Zeits., 15, 528–534, 558–563, 1914.

anode and cathode connections to the tube are interchanged, that is, if the aluminum electrode is made the anode and the platinum electrode the cathode, a good Righi column is obtained, showing that the *shape* of the electrodes and their *position* with respect to the long tube have very little to do with the production of the Righi effects, when these effects are produced with a spark gap.

When the gap is closed, there is, of course, no Righi column, and the discharge through the side tube is continuous. If the gap is now opened and gradually widened, the current through the circuit decreases, remaining continuous, until it has fallen to a certain definite value, when, for a certain critical length of the gap, it becomes periodic. When this happens a short Righi column appears in the long tube. As the length of the gap is further increased the length of the Righi column increases until it fills the whole tube. When the gap becomes too long, the discharge goes out altogether. The frequency of interruption of the discharge decreases as the length of the gap increases. Using 2,700 volts and 800,000 ohms in the circuit shown in Fig. 5, the Righi column appeared when the gap was about .1 mm. long, was brightest at about .3 mm., and went out at about .4 mm.

In most of my experiments, with the spark gap, I used about 2,700

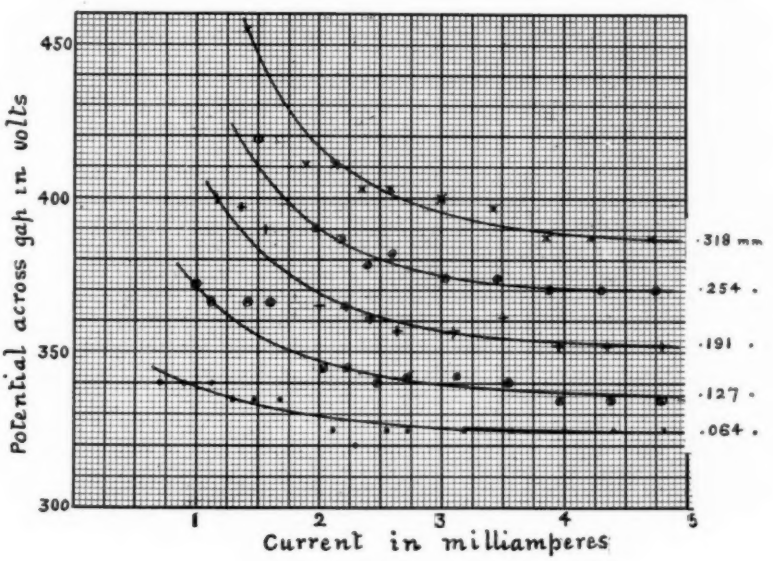



Fig. 6.

volts in the circuit. For this voltage, the Righi effects appear best at a pressure of about .16 mm. of mercury. For 2,100 volts, they begin to appear at about .2 mm., are brightest at about .13 mm., and disappear at about .05 mm.

I examined the charges on the inside of the long tube, when using a spark gap to produce the Righi effects, just as I did when using a magnetic field to produce them, and I got the same results as before. Also, in this case, I found that if the discharge was stopped before the spark gap was opened, there being therefore no luminosity in the long tube, I got a residual positive charge.

From these results, it now appeared that the Righi effects were not dependent directly upon the action of a magnetic field, and were probably produced by the periodic interruption of the current through the tube. It therefore became necessary to study the properties of the spark gap,

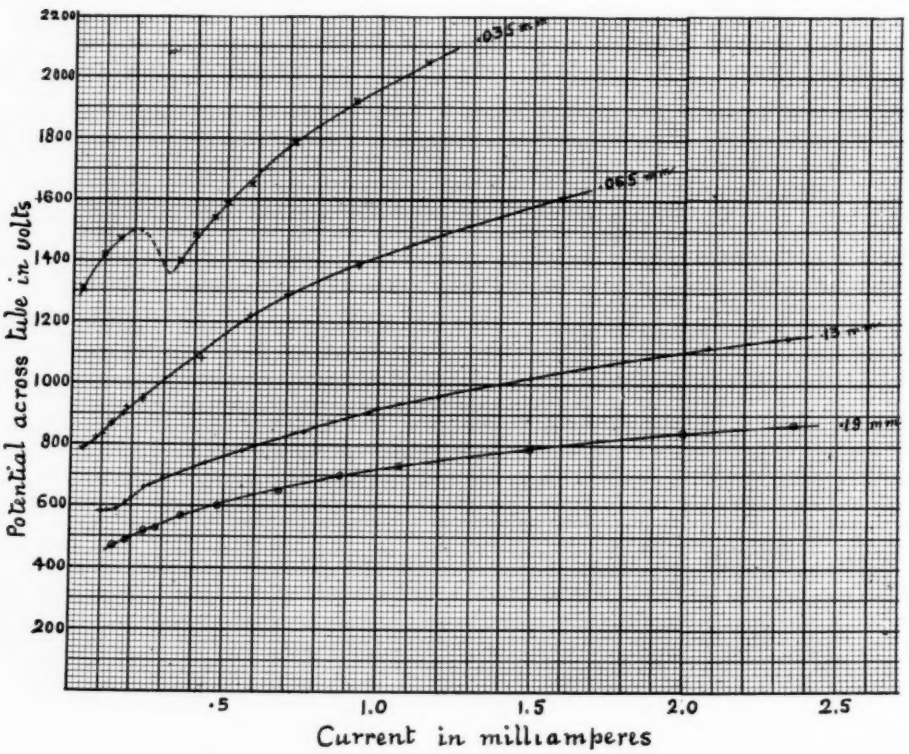



Fig. 7.

and to determine the volt-ampere characteristics both of the spark gap and the Righi tube. The discharge across the gap was examined with both a spectroscope and a microscope and was found to be a true glow discharge, for all lengths, and not an arc discharge. The spectrum observed was, in all cases, the spectrum of nitrogen and not that of the metal of the electrodes. The volt-ampere characteristics for the spark gap, for different lengths of gap, are shown in Fig. 6; and for the tube, for different pressures in the tube, in Fig. 7.

Riecke,[1] Stark[2] and Earhart[3] have investigated the characteristic curves for the *glow discharge* for Geissler tubes of special form; and Stark[4] and Malcolm and Simon[5] for spark gap discharges; and my curves agree, in a general way, with those obtained by them. The shape of the characteristic will, of course, depend upon the shape of the tube, and the form of the gap, respectively. The characteristics, as has been shown by Kaufmann,[6] completely describe the behavior of the gap and the tube in the circuit, and enable us to tell whether the conditions in the circuit are stable or not for a given current by the use of his criterion of stability

$$R > -\frac{\partial E}{\partial i},$$

where $E$ is the potential across the gap, or tube; $i$, the current through it, and $R$ the external resistance in the circuit.

In the circuit shown in Fig. 5, for 2,100 volts and about 500,000 ohms, the discharge becomes discontinuous, or unstable, when the current falls to about 1.8 milliamperes. Now an inspection of Fig. 7 shows that for this current the characteristic for the tube is a rising one, and therefore the conditions are stable so far as the tube is concerned, and the interruption is not due to the tube. An inspection of Fig. 6, on the other hand, shows that for 1.8 milliamperes the characteristic for the gap is a falling one, and that therefore the instability might be due to the gap. However, calculation shows that this possibility must be dismissed as the resistance in the circuit is very large, varying from half a million to a million ohms, and the slope of the characteristic curve, $\partial E/\partial i$, for 1.8 milliamperes is small compared with it, about 30,000 $\frac{\text{volts}}{\text{amperes}}$. Some other cause must therefore be found for the periodic extinction of the spark, as the falling characteristic of the gap is not *alone* sufficient to cause it.

A long series of experiments seems to show that the interruption of the discharge across the gap is due to two facts, (1) that the Righi tube has an appreciable electrostatic capacity, and (2) that the characteristic of the gap is a falling one. The capacity of the tube, apparently, produces an electrical oscillation in it, and this oscillation combined with

[1] Ann. Phys., 4, 592–616, 1901.

[2] Phys. Zeits., 3, 274, 275, 1902; Ann. Phys., 12, 1–30, 1903.

[3] PHYS. REV., 1, 85–95, 1913.

[4] Phys. Zeits., 4, 535–537, 1903.

[5] Phys. Zeits., 8, 471–481, 1907.

[6] Ann. Phys., 2, 158–179, 1900; see also Barkhausen, Problem der Schwingungserzeugung, Leipzig, 1907.

the falling characteristic of the gap produces the interruption. It is found that if the tube is taken out of the circuit, the discharge across the gap is continuous for all lengths of the gap; and also that if the gap is taken out of the circuit, the discharge through the tube is continuous. The periodic interruption of the current flowing through the circuit is therefore due to the combined action of the gap and tube in series.

Light is thrown upon the problem by the fact that the tube can be replaced, in the circuit, by a *high resistance shunted by a capacity*; thus the Righi tube shown in Fig. 1 may be replaced, approximately, by a high non-inductive resistance, $R_1$, of half a million ohms, shunted by an air condenser, $C$, having a capacity of 93 cm., as shown in Fig. 8. The current flowing through the circuit and the periodicity of the discharge were then about the same as when the tube was present.

Further experiments showed that in order to make the discharge across the gap discontinuous, the capacity must be connected directly to the gap. It will not make the discharge interrupted if a high resistance is placed between it and the gap. For instance, a wire, $O_2O_5$, Fig. 9,

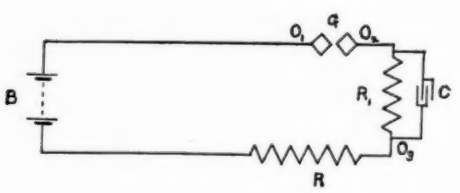



Fig. 8.

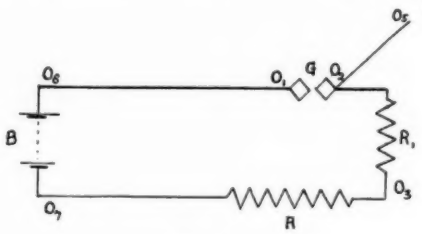



Fig. 9.

800 cm. long and .062 cm. in diameter, and having a capacity free in air of 39 cm., when attached at $O_2$ made the discharge discontinuous. If it was attached at $O_3$, $R_1$ being a high resistance of half a million ohms, it did not make the discharge discontinuous. Also if attached at $O_1$, it did not make the discharge discontinuous. Further it was found that the capacity must be attached between the gap and the high resistance. If it is attached at $O_2$ and the high resistance is placed between $O_1$ and $O_6$ the interruption is not produced. Thus it is evident that they must both be on the same side of the gap. It is however indifferent whether they are attached to the anode or the cathode of the gap.

If a neon tube is touched to any part of the wire $O_2O_5$ it glows. This fact, I think, indicates that electrical oscillations are set up in the system $O_5O_2O_1$ by the charging of the wire $O_2O_5$.

The wire, attached at $O_2$, can be replaced by a capacity connected *across* the gap; the discharge then becoming discontinuous as before.

Elaborate experiments using capacities across the gap and at different points of the circuit seem to show that the periodic extinction of the discharge is produced as follows.

When the key of the circuit is closed, the system formed by the capacity $O_2O_5$ and the battery wires $O_1O_6$ charges up until the potential across the gap is high enough to make the discharge pass across it. The time that this takes will depend upon the time constant, $CR$, of the circuit, where $C$ is the capacity of the wire or condenser, and $R$ is the resistance of the circuit, and also upon the length of the spark gap. Since R is large, this time will not be very small, but will be of the order of the observed period of the discharge, namely a few ten-thousands of a second.

The resistance of the circuit is so great, and its inductance, .000112 henry, so small, that oscillations cannot take place in it as a whole; but oscillations do take place in the free wire and the gap, $O_5O_2O_1$, Fig. 9; the wire going to the battery, $O_1O_6$, acting as an earth. In the circuit used, the battery wires going from the double switch, *kk*, Fig. 3, to the battery two floors above were parallel to each other, about 6 inches apart, and 50 feet long. The oscillations are shielded from the rest of the circuit by the high resistances $R$ and $R_1$. The distribution of the oscillations around the circuit was determined roughly by touching a neon tube to different parts of it.

The oscillations resemble the well-known Poulsen and Wien oscillations but differ from them in the fact that in these the supply circuit necessarily has large inductance and small resistance, whereas the Righi circuit has a very small inductance and a very large resistance.

The discharge is put out in the following manner: The condenser, or wire, charges, the spark jumps across, and a damped oscillating current is set up across the gap. At the same time the battery $B$ tends to build up a steady current of a few milliamperes or less. The growing steady current, and the damped oscillating current, will be superposed upon each other. Every time the resultant current approaches zero it will be extinguished on account of the falling characteristic of the gap. If the oscillating potential across the gap, due to the condenser, is great enough, the discharge will light up again across the gap in the opposite direction. When, however, the oscillating potential is no longer great enough to do so, the discharge will stop altogether. The action will then be repeated, and the audible periodicity of the discharge will be produced. The same action has already been described by Barkhausen[1] for the case of Wien oscillations.

[1] Loc. cit., § 46, pp. 94, 95.

The current oscillations were actually observed with a Braun tube and a rotating mirror, for the case in which .095 microfarad and 3.01 henries were connected in series across the gap, the period being, therefore, approximately $3 \times 10^{-3}$ sec. A strongly damped train of from 1 to 7 half oscillations for every interruption of the spark could easily be observed. The damping as observed in the mirror, or as determined by the position of the maxima on the fluorescent screen of the Braun tube appeared to be *linear* within the errors of observation; the tops of the peaks, in one case, being 2.70, 1.75 and .90 cm. above the point of rest, and 1.75 and .90 cm. below it. I tried to take photographs of the images on the fluorescent screen, but the light was not intense enough.

A calibration of the Braun tube showed that the deflection of 2.70 cm. corresponded to a maximum amplitude of the oscillating current of .33 amperes. The steady current across the gap, when no inductance and capacity were connected across it, was only about .002 ampere. This shows that quite a small capacity would make the discharge interrupted.

I found it to be generally true that if a capacity is put around a spark gap across which a *glow* discharge is passing, the discharge will become discontinuous if the capacity is *great enough*.[1] If the capacity is not great enough, the interruption of the discharge will not take place. For instance, in the case of Fig. 9, the wire $O_2O_5$ must be about 400 cm. long to produce the interruption. Such a wire has an electrostatic capacity of about 1.25 cm. And for the case of Fig. 8, the air condenser, $C$, had to have a capacity of at least 2 cm. before it would make the discharge periodic.

There are in these effects two periodicities to be considered, first, the audible periodicity produced by the separate spark discharges and, second, that of the electrical oscillations producing the interruption of the discharge.

The audible periodicity is approximately equal to the time that it takes for the gap to charge up to the sparking potential. This will depend upon the length of the gap. A curve showing the minimum sparking potential across the gap that I used is shown in Fig. 10.

If we neglect the effect of ionization in the gap, the rise of potential across it for the circuit shown in Fig. 9 will be given by the equation

$$V = V_0(1 - e^{-(t/RC)}), \tag{1}$$

where $V_0$ is the electromotive force of the battery, $R$ is the high resistance in the circuit, and $C$ is the capacity of the wire, or of the condenser around the gap. And the period of the audible periodicity will be given,

[1] See Hittorf, Ann. Phys., 20, 719–726, 1883; Lehmann, Ann. Phys., 56, 333–336, 1895.

approximately, by

$$t = CR \log_e \left( \frac{V_0}{V_0 - V} \right), \tag{2}$$

where $V$ is equal to the minimum sparking potential across the gap. $V$ for a given length of gap can be determined from Fig. 10. If $V$, $V_0$, $R$ and $C$ are known, the audible period can be determined.

I observed that as $R$, $C$, or the length of the gap, was increased the audible period was increased. $t$ was calculated from the known values

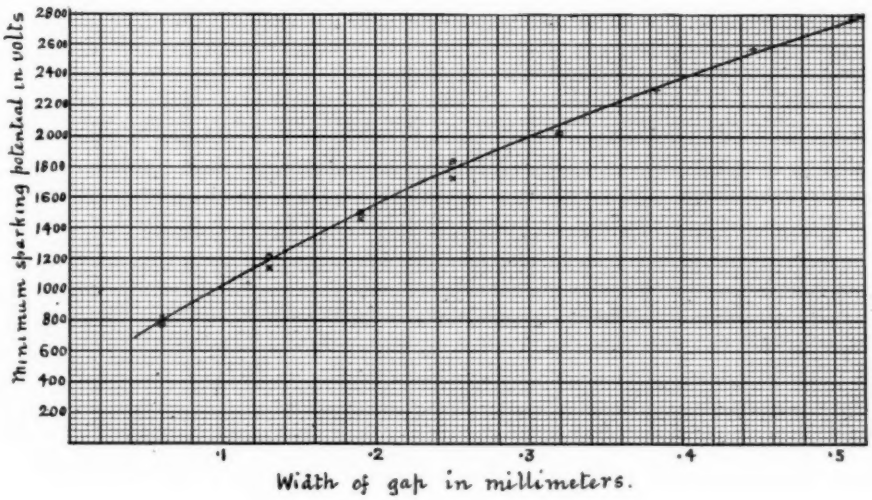



Fig. 10.

of $C$, $R$, $V_0$ and $V$, and the values obtained were of the order of the audible period.

If we assume that the discharge across the gap goes out when the oscillatory current is equal and opposite to the steady current, since the maximum value of the oscillatory current, neglecting damping, is

$$V \sqrt{\frac{C}{L}}, \tag{3}$$

where $L$ is the inductance of the circuit, and the value of the steady current is

$$\frac{V_0}{R}, \tag{4}$$

we shall have as the condition for the production of an interrupted discharge

$$V \sqrt{\frac{C}{L}} \geqq \frac{V_0}{R}.$$

Solving for $C$, we get

$$C \geqq \left(\frac{V_0}{V}\right)^2 \frac{L}{R^2} \tag{5}$$

as the equation for the minimum value of the capacity which must be shunted around the gap, or attached to the cathode side of the gap, to make the discharge an interrupted one.

$V$, the minimum sparking potential, shown in Fig. 10, is related to the length, $l$, of the gap approximately by the formula

$$V = k_0 + kl, \tag{6}$$

where $k_0$ and $k$ are constants.

Experiments confirmed (5) except that the value of the inductance, $L$, of the oscillating circuit seemed to have little effect upon the value of $C$. If anything, the experiments indicated that the greater $L$ the less $C$ had to be. This may perhaps be due to the fact that the minimum value of $C$ is always very small, and that any inductance that can conveniently be used has a relatively large distributed capacity. So that, when we add inductance to the circuit we add more or less capacity.

The experiments described show, I think, that when the *Righi effects* are obtained without a magnetic field, with a spark gap in the circuit, they may be explained by assuming that the capacity of the long tube and the instability of the spark gap produce a periodic interruption of the current. There is thus produced an alternate charging and discharging of the long tube, the negative column in the long tube corresponding to the charging, and the positive column to the discharging. An actual electrical oscillation takes place in the long tube, made visible by the light resulting from the ionization of the gas. The virtual anode is the region in which the charging and discharging streams are both visible. These two streams probably occur at the other points along the tube, but on account of lack of luminosity at these points, are not visible.

The action is probably as follows: When the double switch leading to the battery is closed, the potential across the gap rises until there is a discharge across it. When this happens, there is a sudden flow of current through the side tube, and the long tube charges up positively. There is then a reaction of the current in the side tube on account of its capacity and inductance, and a flow of current in the opposite direction. This reduces the total current flowing across the gap, and on account of the instability of the gap, the current through it becomes extinguished. It is possible that there may be more than one oscillation of the current in the long tube, but I do not think that there is. The oscillation can

only take place in the conducting system $DTO_2O_1B$, Fig. 5. It cannot extend into the portion $O_3R$ on account of the high resistance $R$. The luminous column, however, in the side tube from $A$ to $M$, Fig. 2, when under the action of a transverse magnetic field, is seen to be always pressed against one side of the tube only. If there were an oscillating current from $A$ to $M$, there would be *two* columns, pressed against opposite sides of the tube. Therefore, I think that there is probably only one oscillation in the long tube, a flow in and a flow out.

It is well known that alternations of current of low frequency can exist in Geissler tubes,[1] and also that if the luminous column of the discharge is subjected to a transverse magnetic field it will be separated into two columns bent towards opposite sides of the tube, corresponding to the opposite currents. The question arose as to whether this would be true for alternations of current in the long tube where the alternations, if they exist, must be of very high frequency, probably several millions a second. I found by experiment that such oscillations of current in Geissler tubes *are* divided into two streams by a transverse magnetic field.

A tube 10 cm. long and 1.5 cm. wide had platinum wire electrodes exactly alike and 7 cm. apart sealed into its ends. It was connected in series with a condenser and the secondary of a small induction coil, and subjected to the action of a transverse magnetic field. A double positive column was obtained, the two columns being pressed against opposite sides of the tube. The two columns were, in many cases, equally bright, indicating, I think, that there must have been several oscillations for each break of the primary circuit. The capacity was varied from $2 \times 10^{-6}$ mf. to 1.25 mf. When the capacity was increased to 2.5 mf., only one column was present. For 1.25 mf. two columns were present, but one was much less bright than the other. When the tube was connected directly to the terminals of the secondary without any condenser in series with it, there was only a single column when acted upon by the magnetic field.

To examine the action for very high frequencies, a wire 70 cm. long was attached to each side of the tube, and a spark several millimeters long was caused to jump from the secondary of the induction coil to the end of one of the wires, the end of the other wire on the other side of the tube being free in air. The tube then glowed, and when subjected to the magnetic field showed a double column. Since the tube with its attached wires must act as a linear oscillator, or a Hertz oscillator,

[1] Thomson, Phil. Mag., 32, 321–336, 445–464, 1891; Lehmann, Verh. d. Naturwissensch. Ver. Karlsruhe, 15, 69–71, 1902.

the frequency must be very high. In this case, since the wires were each 70 cm. long, the frequency was probably of the order of $10^8$ a second.

The fact that, to get the best results with the Righi tube, the gap must be placed next to the anode of the tube, and the high resistance next to the cathode, is probably explained by the apparent high resistance of the region around the cathode,[1] shown by the high value of the cathode fall of potential. On the other hand, the region of the positive column behaves as though it had a relatively small resistance. If then the gap is placed next to the cathode of the tube, the region of the cathode will act like a high resistance between it and the long tube, and tend to prevent oscillations from being set up in the long tube.

### 6. A Third Way of Producing the Righi Effects.

While experimenting on the pressure most favorable for the Righi effects, I found that they could be produced without either a magnetic field or a spark gap, if the pressure in the tube was made low enough. For instance, using about 2,100 volts and 500,000 ohms in series with the tube, I got a well-marked column with all the Righi effects when the pressure of the air was reduced to .035 mm. The current through the tube at this pressure was about .2 milliampere. Above this pressure, the discharge through the tube was continuous. Below it, it became discontinuous and the Righi column appeared.

The appearance of periodicity at low pressures is probably explained by the fact that at these pressures the current is very small, and that the volt-ampere characteristic for the tube becomes a falling one for very small currents. The discharge would then become unstable for a certain value of the current according to Kaufmann's criterion. The characteristic for the tube for .035 mm. of pressure, shown in Fig. 7, would suggest that such is the case. The curves given by Riecke[2] and Earhart[3] also show that when the current is very small the characteristic becomes a falling one.

In this case it will be observed that the Righi effects are produced not only without a magnetic field, but even without the presence of a spark gap. So apparently the capacity of the tube and its periodic charging are the only elements that are necessary to produce the Righi effects.

The explanation for this case is probably the same as for the case in which there is a spark gap, except that in this case the instability of the

[1] See Reiger, l.c., p. 35; Lehmann, l.c., pp. 71–73; Wiedemann and Schmidt, Ann. Phys., 62, 462, 1897.

[2] Loc. cit., p. 596.

[3] Loc. cit., pp. 88–91.

discharge through the tube takes the place of the instability of the discharge across the gap.

## 7. CONCLUSIONS AND SUMMARY.

Since I have shown that the Righi effects can be produced without any magnetic field by inserting a spark gap in the circuit, and, if the pressure is low enough, without either magnetic field or spark gap, it may be assumed that the magnetic field does not play a fundamental part in the production of the effects.

In Righi's case, the magnetic field apparently interrupts the discharge. There then follows a rising of the potential across the gap until the discharge takes place again. At this instant, there is a charging of the long tube, and this results in an electrical oscillation in it, in virtue of its capacity and inductance. It may consist of a single oscillation or of several; probably of a single one.

My conclusions therefore may be summarized as follows. The effects observed by Righi in a special form of Geissler tube, and believed by him to arise from a new kind of ray, called by him the magnetic ray, are not due to any new form of ray, but are due to the periodic charging and discharging of the tube.

In support of this view, I have shown:

1. That the long horizontal extension of the Righi tube has an appreciable electrostatic capacity.

2. That the principal action of the magnetic field is to periodically interrupt the discharge through the tube.

3. That for each interruption, charges appear in the long tube, an initial positive charge and a residual negative charge.

4. That the Righi effects can be produced without any magnetic field, by using a suitable spark gap in the circuit.

5. That the current through a circuit of high resistance and small inductance, containing a battery and a spark gap, can be changed from a steady to an interrupted current by attaching a capacity to one of the terminals of the gap; and that this periodic interruption of the current is due (1) to an oscillation across the gap produced by the attached capacity, and (2) to the negative slope of the volt-ampere characteristic of the gap. A theory is given for this action.

6. That alternations of current of very high frequency can exist in a Geissler tube, and that their presence is made evident by a *double* luminous column when the tube is subjected to a transverse magnetic field.

7. That the Righi effects will be produced without either magnetic field or spark gap if the pressure in the tube is made low enough.

In conclusion, I wish to thank the authorities of Clark University, and Professor A. G. Webster in particular, for placing all the resources of the physical laboratory at my disposal during the course of this investigation.

CLARK UNIVERSITY,
WORCESTER, MASSACHUSETTS,
November 15, 1916.

# THE SINGLE-LINED AND THE MANY-LINED SPECTRUM OF MERCURY.[1]

BY T. C. HEBB.

FRANCK and Hertz have shown[2] that under certain conditions a single-lined spectrum of mercury can be obtained. They found that one of the necessary conditions was that the vapor of mercury be bombarded by electrons having a speed equal to or greater than that produced by falling through 5 volts. As they had already shown that the ionizing potential of mercury was 4.9 volts they were led to an application of the relation $Ve = h\nu$ where $h = 6.56 \times 10^{-27}$ erg. sec. and $\nu$ is the frequency of the single-lined spectrum $\lambda = 2536.72$ Å. U. The experiments of Franck and Hertz are exceedingly suggestive and would appear to offer new light on the structure of the atom.

These results have since been repeated and extended by McLennan and Henderson.[3] They found that they could get the single-lined spectrum of mercury for any potential difference between 5 and 12.5 volts, but that with a potential difference below 5 volts they obtained no spectrum and that with a potential difference of 12.5 volts or higher they always obtained the many-lined spectrum of mercury. In order to explain the occurrence of the many-lined spectrum at 12.5 volts they suggest that possibly a new type of ionization occurs at this point. They have further shown that they can obtain single-lined spectra of both cadmium and zinc. In a still later article McLennan has shown[4] that he can also obtain a single-lined spectrum of magnesium. As the ionizing potentials of cadmium, zinc and magnesium have not been determined directly the authors apply the above quantum relation $Ve = h\nu$ in order to determine the value for each of the metals.

The results embodied in this paper are considered worthy of publication even though the work is incomplete. The writer had occasion to spend a limited time at the Ryerson Physical Laboratory during the spring of 1916 and at a suggestion from Professor Millikan attempted

[1] This and the following paper were presented before the American Physical Society at the Chicago meeting, December 1, 1916.

[2] Deutsch. Phys. Gesell. Verh., 16, 11, pp. 512–517, June 15, 1914.

[3] Roy. Soc. Proc., 91, pp. 485–491, Aug. 2, 1915.

[4] Roy. Soc. Proc., 92, pp. 305–312, Mar. 1, 1916.

to repeat the experiments of Franck and Hertz and of McLennan and Henderson and if possible throw light on the relations of the single-lined and the many-lined spectrum of mercury.

The results to be stated later were obtained with the following apparatus:

*A* (Fig. 1) was a quartz tube 20 cm. in length, 1.9 cm. internal diameter and 2.1 cm. external diameter. *B* and *C* were iron caps which were joined to the tube with Khotinsky cement. Through *B* passed a glass tube which was connected to the air pump and through which also passed the wire leading to the platinum anode *D*, mounted on the end of the glass tube. Passing through *C* were two glass tubes through which passed the iron electrodes which held the cathode *E*. The latter was attached to the iron electrodes with clamps. The glass tubes were fastened to the iron caps *B* and *C* with Khotinsky cement. The above arrangement of apparatus made it possible to renew the platinum cathode and also to vary the distance between the anode and cathode. The central part of the tube was placed in an asbestos gas-heated furnace with a sheet-iron bottom. The collimator of the spectrograph passed through a hole in the furnace and the slit was placed directly against the quartz tube. *B* and *C* were surrounded with copper water jackets and the iron electrodes at *F* were cooled with a stream of air in order to prevent the cement from softening. With this arrangement and with the iron electrodes 1/8″ in diameter it was found possible to use 20 amperes through the cathode and heat the furnace with a gas flame and not have the cement connections soften even after hours of operation. It should be stated that the diagram shows a horizontal section and that the mercury usually stood directly under the anode and cathode. To prevent the heat from passing along the mercury and melting the cement at *C* or *B* the tube was inclined so that the mercury always stayed in the *C* end of the tube and at *G* there was placed an iron stop which prevented it from reaching the end of the tube. The cathode consisted of a piece of platinum .0025 cm. thick and about 1.5 cm. long. The width varied in different cases from 2 mm. to 4 mm. and the current to heat it from 10 to 20 amperes. The platinum was either used bare or coated with barium oxide. The photographs were taken with a small quartz spectrograph. The slit was directed towards some point between the anode and cathode. The lens was of such size that it gathered light from all points between the anode and cathode if they were placed 1.5 mm. apart.

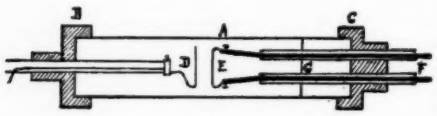



Fig. 1.

With the apparatus outlined as above one of the first difficulties encountered was due to the occurrence of the many-lined spectrum. It was found almost impossible to make exposures at voltages used by other experimenters for any considerable length of time without having an arc strike and then the plate showed the many-lined spectrum. In order to obviate the striking of the arc the potential difference between the electrodes was lowered step by step, until finally at a little under 5 volts it was found that it was possible to photograph for hours without the appearance of the many-lined spectrum. It thus appears from these experiments that the many-lined spectrum can be obtained for any potential difference above 5 volts. As the other experimenters, as well as the writer, have obtained the single-lined spectrum above 5 volts we have the interesting fact that both the single-lined and the many-lined spectra of mercury may be obtained above 5 volts. McLennan and Henderson (l.c.) obtained the single-lined spectrum for potential differences as high as 12.5 volts. The writer has been able to obtain it for potential differences as high as 21.5 volts. No attempts have been made, as yet, to obtain it at higher voltages, but there seems to be no reason why it could not be obtained.

The fact that the many-lined spectrum of mercury can be produced for the same potential differences for which the single-lined spectrum has been obtained raises the question as to whether there is such a thing as a true single-lined spectrum of mercury above 5 volts. Or to put it another way can the radiation $\lambda = 2536.72$ Å. U. be produced above 5 volts without the appearance of any of the other radiations of mercury? The writer restricts the question to a potential difference greater than 5 volts for reasons which will appear later. Further, if either the single-lined spectrum or the many-lined spectrum can be produced independently of one another above 5 volts, what are the conditions necessary in order to bring out either the one or the other? The spectrogram labelled I. in the accompanying plate is submitted relative to the above questions. This spectrogram was taken under the following conditions. The maximum and minimum potential differences between anode and cathode were 9.0 and 8.4 volts. The distance between the anode and cathode was between 2 and 3 mm. The temperature immediately outside of the tube was about 160° C. The slit of the spectrograph was arranged so that part of it received light from between the anode and cathode and part from below them. The time of exposure was 3 hours. That part of the plate which received light from between the anode and cathode shows the many-lined spectrum with the line $\lambda = 2536.72$ Å. U. relatively stronger than the others, whereas, that

part of the plate which received light from below the anode and cathode shows only the line $\lambda = 2536.72$ Å. U. Further, the lines other than the line $\lambda = 2536.72$ Å. U. have blunt ends showing no tapering effect. It is hoped that this appears in the reproduction. In as much as the lines other than the line $\lambda = 2536.72$ Å. U. show no tapering effect, this spectrogram seems to indicate that the single-lined spectrum can be produced without any of the other radiations being present. The spectrogram also seems to show that the occurrence of the single-lined spectrum or the many-lined spectrum depends upon the density of the electronic discharge. From the bluntness of the lines it would also appear as if there were a minimum density of electronic discharge which was necessary in order to produce the many-lined spectrum. It should be pointed out that no variation in the density of the mercury vapor could be present to explain the above spectrogram.

Another spectrogram bearing on the above questions is shown in II. of the plate. The maximum and minimum potential differences in the case of this spectrogram were 21.5 and 20 volts. Ordinarily such a voltage would produce an intense arc, but in this case the cathode consisted of uncoated platinum which had been heated in a good vacuum for 30 or 40 hours. As a consequence the electronic discharge from it was very small. In no other way did the experiment differ materially from others where strong spectra were obtained. The spectrogram shows only the line $\lambda = 2536.72$ Å. U.

Spectrogram III. also bears on the above questions. It was taken under the following conditions. The maximum and minimum potential differences were 5.5 and 4.7 volts. The cathode was coated with barium oxide. The distance between the anode and cathode was about 3 mm. The temperature immediately outside the tube was about 165° C. The time of exposure was 4 hours. The plate shows the line $\lambda = 2536.72$ Å. U. very plainly and some of the other stronger lines of the many-lined spectrum faintly. The writer is unable to decide whether the many-lined spectrum in this case came in slowly or whether it was produced by some sudden increase in the density of the electronic discharge.

Although the above spectrograms are conflicting in their evidence, the writer is of the opinion that it is possible to produce the radiation $\lambda = 2536.72$ Å. U. without any of the rest of the radiation of mercury being present. Further, in order to produce the single-lined spectrum for potential differences greater than 5 volts, it seems to be necessary to keep the density of the electronic discharge below a certain minimum value and the greater the potential difference used the smaller must this minimum value be.

In endeavoring to locate the lowest potential difference at which the many-lined spectrum was produced, it was found possible to obtain the single-lined spectrum for potential differences considerably below the minimum as obtained by the other experimenters. Spectrogram IV., for instance, was obtained with maximum and minimum potential differences of 2.5 and 1.9 volts. The plate in this case was exposed 3 hours. Slight traces of the line were seen on a plate exposed 4 hours with maximum and minimum potential differences of 2.0 and 1.4 volts. These results raise a question relative to whether a potential difference across the electrodes is necessary in order to produce the single-lined spectrum. The writer is not able to answer this question. Whatever may cause the single-lined spectrum, it decreases in intensity very rapidly with decreasing voltage. In order therefore to get it at these low voltages every means for increasing the intensity of radiation must be used. But even after the radiation has been made as intense as possible a point is reached where the fogging of the plate masks the effect looked for. Spectrogram V. shows how intense the radiation can be obtained for maximum and minimum potential differences of 4.1 and 3.5 volts. The plate in this case was only exposed 2 hours. A comparison of spectrograms IV. and V. will give an idea relative to the decrease in intensity of the radiation $\lambda = 2536.72$ Å. U. with the decrease in voltage.

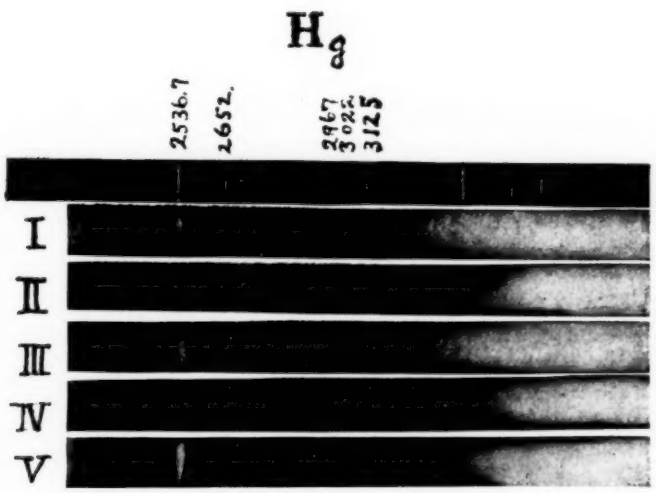
The production of the single-lined spectrum of mercury at such low voltages as recorded above seems to cast doubt on the deductions of Franck and Hertz. These experimenters came to the conclusion that ionization was necessary in order to produce the single-lined spectrum and as ionization occurs at 4.9 volts for mercury they did not believe that the single-lined spectrum could be produced below that voltage and their experimental results seemed to bear out their theory. If ionization is necessary to produce the single-lined spectrum then we must conclude that the electrons leave the hot body with a velocity greater than has been supposed. The velocity of emission would have to be of the order of 2 or 3 volts. Richardson and Brown have determined the velocity of emission from platinum and from platinum coated with an oxide.[1] In the case of platinum they never found the velocity greater than 0.6 volt and in the case of the oxide-coated platinum the greatest velocity obtained was 1.2 volts.

As these results seemed to point to an explanation it was thought worth while to try and measure the velocity of emission right in the tube under observation. For this purpose the anode was connected to the one pair of quadrants of an electrometer with the other pair grounded.

[1] Phil. Mag., Vol. 16, p. 353 (1908).

The negative end of the cathode was joined to the earth through a variable potential difference, so that its potential could be varied with respect to the earth. If the hot cathode gave out only electrons then by raising its potential a value could be reached for which no charge could pass to the electrometer. As is well known both positive and negative particles are given out from a hot body. With the two streams of particles coming from the cathode it was found that by raising the potential of the negative end of the cathode a value was reached which gave no deflection of the electrometer. This means that the positively charged particles forced out by the potential of the cathode just balanced the negatively charged particles which, owing to their initial velocity, overcame this same potential. If the potential of the cathode was raised above this point the electrometer acquired a positive charge and if it was lowered below this point it acquired a negative charge. Were the positive stream to be eliminated then the potential would have to be still higher in order to prevent the electrometer from acquiring a charge. In the case of the writer's apparatus it was found that twenty or thirty hours of heating and pumping were not sufficient to greatly reduce the positive stream. This was probably due to the great amount of matreial from which it was necessary to drive the occluded gases. In only one case was the positive stream reduced to small proportions. This was for the case of pure platinum. As time was limited no effort was made to overcome the difficulties. The general results obtained, however, were these. With pure platinum for both anode and cathode the potential necessary in order to get zero deflection of the electrometer after 20 or 30 hours of heating was about .25 volt. Had there been no positive stream this value would have been raised. This result agrees quite well with that of Richardson and Brown for uncoated platinum. With platinum coated with barium oxide the potential necessary to get zero deflection of the electrometer was very much greater than for pure platinum. After long heating and pumping a value as high as 2.3 volts was obtained. As there was still a large positive stream this value would have to be raised considerably, perhaps to 3 volts, in order to stop the negative stream. To conclude from the above results that the velocity of emission is of the order of 3 volts would, it appears, be unsafe for there are a number of uncertain factors. The results, however, are suggestive and the phenomena need further investigation.

As the velocity obtained for pure platinum could not be much in doubt an attempt was made to find the potential difference for which the single-lined spectrum appeared for uncoated platinum electrodes which had been heated in a vacuum for 30 or 40 hours. It was found, however, that no trace of a line appeared for voltages in the vicinity 4.9.




HEBB AND MILLIKAN.

Even potential differences as high as 20 volts gave only a very weak line after 4 hours' exposure. (See II. of the plate.) This was no doubt due to the smallness of the electronic discharge. It is evident that the question relative to whether ionization produces the single-lined spectrum is still unanswered.

Another result obtained in connection with these experiments which needs explanation is the fact that an arc was made to operate at a potential difference as low as 3.2 volts. The arc was never found to strike at a potential difference lower than 5 volts but after it was once in operation there were times when the maximum voltage across it was as low as 3.2 volts. The spectrum from it then showed the many-lined spectrum of mercury. If a velocity of emission of the order of 2 volts be assumed for the electrons then the arc at these voltages can be explained, otherwise it must be assumed that the arc itself produces the nceessary ionization to keep itself in operation. If the velocity of emission of the electrons is of the order of 2 or 3 volts then it appears that the arc should be capable of striking at 2 or 3 volts if ionization is all that is necessary. This result has not been observed.

The experiments outlined in this paper have shown: (1) that the mercury arc can be made to strike at any potential difference greater than 4.7 volts, thus producing the many-lined spectrum; (2) that the arc having been made to strike will operate as low as 3.2 volts, producing the many-lined spectrum. This needs a special explanation for the production of its ions unless it is shown that the velocity of emission is sufficient to account for the discrepancy; (3) that the single-lined spectrum can be obtained for all potential differences between 2.5 and 21.5 volts. In order to explain its production below 5 volts it must be assumed that there is a large velocity of emission or else the idea that ionization is necessary for the emission of light must be abandoned. It may be that ionization is necessary to produce the many-lined spectrum but that a jarring effect is all that is necessary to produce the single-lined spectrum; (4) that whether the many-lined or single-lined spectrum of mercury appears above 5 volts depends upon the density of the electronic discharge.

The writer wishes to express his gratitude to the staff of the Ryerson Physical Laboratory, University of Chicago, for placing at his disposal the facilities for doing these experiments, and especially to Professor Millikan, whose interest in this subject inspired the work.

Since doing the above experiments the writer has become associated with the University of British Columbia and expects to continue this work.

University of British Columbia, Vancouver, B. C.

# THEORETICAL CONSIDERATIONS RELATING TO THE SINGLE-LINED AND THE MANY-LINED SPECTRUM OF MERCURY.

R. A. MILLIKAN.

AT the meeting of the Physical Society of Chicago in November, 1914, I raised a question as to the reality of the so-called one-line spectrum which had seemed to appear in Franck's and Hertz's experiment. I pointed out that if we took the point of view of the pure experimentalist, Wood's demonstration of the enormous absorbing and emitting power of mercury vapor for line of wave-length 2536 Å. along with its transparency for other lines means that with increasing volume excitation the relative intensity of these other lines to that of 2536 must continually increase. Otherwise stated, the opacity of the mercury vapor for line 2536 requires that the light of this wave-length which comes out of a tube come only from the surface layers, while the light of other wave-lengths will come from the whole volume. With weak excitation, therefore, line 2536 might be enormously more intense than the other lines and thus appear on Franck's and Hertz's photographic plates, although the other lines might still be present but merely be too weak to be noticed with the exposures used.

On the other hand, from the point of view of Bohr's theory, a single-line spectrum is not to be expected at least at line 2536. This can be seen from the following considerations: It has long been suspected, and indeed was proved more than a year ago by Tait[1] and again quite recently by Goucher,[2] that though when negative electrons are driven through mercury vapor there is inelastic impact at 4.9 volts, there is very little ionization at that point or indeed at any potential less than about 10.4 volts when strong ionization sets in. This means from the point of view of Bohr's theory, that at 4.9 volts an electron is merely displaced from one particular orbit to the next outer orbit and when it returns it emits the longest line of the series corresponding to a return to this particular orbit. Line 2536 should therefore be the longest wave-length in a series, as it is in fact, and none of the shorter wave-lengths

[1] PHYS. REV., VII., 686, 1916.
[2] PHYS. REV., VIII., 561, 1916.

of this particular series should appear at 4.9 volts, but the whole series should appear when this electron is knocked entirely out of the atom. This would occur, let us assume, at 10.4 volts, a potential which corresponds, in accordance with the quantum relation, to the convergence wave-length (1188 Å.) of the series of which line 2536 is the long wave-length head. The many-lined spectrum which should suddenly appear at 10.4 volts should then be merely the whole series of which line 2536 is the member of longest wave-length.

But now, since the normal spectrum of mercury actually has many lines of greater wave-length than 2536, these must, according to the Bohr theory, belong to orbits farther out from the nucleus than that producing line 2536 for its member of longest wave-length, and these lines should always be produced when 2536 is produced. For if in the permanent state of mercury vapor some of its atoms have electrons in these outer orbits, they must be knocked entirely out at potentials which should not be greater and may be much less than 5.5 volts. To see how this follows from the Bohr theory let the subscripts 1 and 2 apply to any two adjacent series, such as the K and L series in X-rays. The Bohr theory then states that

$$\nu_{\beta_1} - \nu_{\alpha_1} = \nu_{\infty_2}.$$

Kossel[1] first published tests of this relation taken from X-ray data. The equation gives also the correct relation between the Lyman ultra-violet series of hydrogen, the Balmer series, and the Paschen infra-red series, since these all fit perfectly the Bohr theory. Even where the simple form of Bohr's theory, which it will be recalled was developed only for a single negative electron rotating about a positive center, does not apply, the foregoing equation should still hold, for it states merely the energy relations which must be satisfied no matter what influences the electrons in the various orbits may exert upon one another.[2] If then, $\nu_{\infty_1}$ and $\nu_{\infty_2}$ represent the frequency limits toward which the series corresponding to two adjacent orbits run, then the energy relations underlying equation 1 may also take the form

$$\nu_{\infty_1} - \nu_{\alpha_1} = \nu_{\infty_2}. \qquad (2)$$

This is merely the well-known Rydberg-Schuster law[3] discovered wholly empirically and tested quite generally for metallic spectra, but now seen to be a necessary consequence of the Bohr theory of the atom.

[1] Verh. d. D. Phys. Ges., 16, 953, 1914.

[2] See Presidential Address American Physical Society on "Radiation and Atomic Structure," PHYS. REV., May, 1917.

[3] Bailey's Spectroscopy, p. 488.

Now the series of which the mercury line 2536 is the long wave-length head runs toward the limit $\lambda = 1188$ Å. Applying equation 2, we obtain

$$\frac{1}{1188} - \frac{1}{2536.7} = \frac{1}{x} \therefore x = 2234.$$

*This is indeed exactly the short wave-length limit toward which one of the well-known mercury series runs*, viz., the series 3125, 2652, 2482, ···, 2234, and according to the Bohr theory it should be the series which corresponds to the orbit just outside that which gives rise to the series of which line 2536 is the member of longest wave-length. The value of $h\nu$ in volts corresponding to this wave-length 2234, is 5.5. If, then, there are electrons in this orbit they should be knocked entirely out at 5.5 volts and the whole series of which line 2234 is the convergence frequency should appear. The longer wave-length members of this series should appear at still lower potentials; if they do not so appear it should be because there are no electrons in these orbits.

But even if in the normal mercury vapor there are no electrons in orbits which are larger than that corresponding to the series whose longest wave-length member is 2536, yet since this line is stimulated it must act photoelectrically, as Van der Bijl[1] has just been urging, on the surrounding atoms, for the photoelectric long wave-length limit of mercury is about 2800 Å. and *the return of these detached electrons, or of others which take their places, should be accompanied by all lines of series corresponding to outer orbits which, according to our hypothesis, are normally empty*. As a matter of fact the lines which do appear faintly but altogether definitely on both I. and III. of the foregoing plate, though less clearly on the reproduction, are 3125 and 2652 of the above-mentioned series and in addition 2967 and 3022, which, as Prof. F. A. Saunders kindly tells me, are members of the same triplet series; this series being

| | | | | Limit. |
|---|---|---|---|---|
| 3650 | 3022 | 2804 | ··· | 2491 |
| 3125 | 2652 | 2482 | ··· | 2234 |
| 2967 | 2534 | 2378 | ··· | 2149. |

In the preceding discussion it has been assumed that the emission of light by mercury is due solely to the partial or complete formation of positive atom-ions. But it is conceivable that the light-emission is in part at least due to the simple stimulation of neutral mercury atoms by corpuscular impact. In this case many lines would be expected to appear simultaneously and at a potential which could scarcely be expected to bear any relation to the value of $h\nu$ for any one of the lines.

[1] Proc. Phys. Soc., Chicago meeting, Dec. 1, 1916.

From whatever point of view, then, I approach the excitation of mercury vapor I can see no reason to expect line 2536 to appear without the lines of longer wave-length. These lines would be expected to be weak if there are few or no electrons in orbits greater than that corresponding to 2536, but they should nevertheless be present. I initiated Dr. Hebb's experiments, therefore, in the hope that they would bring to light these weak lines of longer wave-length than 2536 and accompanying it.

His results seem to me to indicate strongly that these lines are in fact always present along with 2536, their appearance on the photographic plate depending, as is to be expected from Wood's experiments, simply on the volume excitation, or in a given tube, upon the density of the exciting electron stream. Thus in I. of the plate the reason these additional lines appear in the upper half and not in the lower is that, as is well known, the electron stream is a sharply limited bundle outside of which the excitation is too weak to permit the appearance of the weaker lines in the exposure time used. The sharp limits of the cathode beam explain fully the bluntness of the ends, without assuming that a definite density is necessary to stimulate the many-lined spectrum. That these weak lines are present in II. when the P. D. is 20 volts but the electron stream weak, no one, I fancy, would be inclined to doubt.

The experiments on the starting potential of the arc seem to me to demonstrate, first, that this potential depends upon the density of the exciting cathode stream; second, that this potential may be anything above five volts.

Both of these conclusions are in accordance with the foregoing theory. For the striking of an arc merely indicates the rise in the number of positive ions to a certain large value and the consequent concentration of the potential-fall in the neighborhood of the cathode. But with the reciprocal relations which have been postulated between the stimulation of light at 4.9 volts and the photoelectric production of ions by this light, there may exist at any potential in excess of 4.9 volts, the possibility of an indefinite multiplication of positive ions. With a weak cathode stream this possibility will not be realized in the neighborhood of 5 volts, since, in view of the strong absorption by mercury vapor of line 2536 the photoelectrons will be formed close to where the light is stimulated, *i. e.*, close to the anode. The negatives so formed will not then fall through an appreciable potential and hence will not help the original cathode stream in the stimulation of light. The denser the original cathode stream, however, the more positive ions will be formed by the light and consequently the more will the total fall of potential become

concentrated near the cathode, and the sooner will negative photo-electric ions begin to be formed where they can aid the original cathode stream in the stimulation of more light. As soon as this condition ensues there arises the possibility of an indefinite multiplication of positive ions and the formation of an arc.

If the cathode stream is weak then the arc will not appear in the neighborhood of 5 volts. It would, however, be expected to appear at the ionizing potential, namely, at 10.4 volts, since at this point there is in any case a large multiplication of ions. This is where the arc actually did appear in McLennan and Henderson's experiments.[1]

It may be mentioned that the fact that no measurable ionization has ever been observed in mercury vapor at potentials lower than 4.9 volts would signify, from the point of view here presented, that when the mercury atom is in its normal, stable condition, the orbit corresponding to line 2536 is the outermost one which contains any appreciable number of its 80 negative electrons.

Whether the foregoing theory is correct or not there can be no doubt that a mercury arc can be maintained at much less than 10.4 volts, for I have myself held one at 3.5 volts, but in this case as in Dr. Hebb's experiments, I suspect that the contact electromotive force was sufficient to raise the actual energy of impact to 4.9 volts. As further evidence it may be mentioned that Rau[2] observed the appearance of the mercury line 4359 at 8 volts, which is below the value, 10.4 volts, which McLennan found to be the lowest at which any of the lines of the many-lined spectrum could appear.

That the quantum relation holds both at 4.9 volts and at the ionization potential, 10.4, volts is rendered probable by Tait's and by Goucher's experiments, as well as by those of McLennan; for both the observed potential of inelastic impact, and that of ionization seem to agree, within the limits of error of the experiments, with predicted values. The present experiments are not at all at variance with McLennan's but they extend them by showing, first, that an arc may be produced without raising the potential to the ionizing value, and second, that weak lines of longer wave-length than 2536 probably always appear with it. It is only the shorter and shorter wave-lengths *of a given series* which ought to appear successively with successive increases in the applied potential. This prediction is also in line with Rau's results.

RYERSON PHYSICAL LABORATORY,
UNIVERSITY OF CHICAGO.

[1] McLennan and Henderson, Proc. Roy. Soc., 91, 485, 1915, and McLennan and Keys, Proc. Roy. Soc., 92, 591, 1916.

[2] H. Rau, Ber. d. Phys. Med. Ges. Wurzburg, 1914.

# ON THE NUCLEAR CHARGES OF ATOMS.[1]

BY FERNANDO SANFORD.

IN his second paper on "High Frequency Spectra," published in Phil. Mag., XXVII., 703 (1914), Moseley says: "The frequency of any line in the $x$-ray spectrum is approximately proportional to $A(N - b)^2$, where $A$ and $b$ are constants and $N$ is the atomic number."

For the $\alpha$ line of the K-radiation Moseley found $b = 1$, and for the principal line of the L-radiation, $b = 7.4$. Moseley surmised that the significance of $N$ in these equations might be explained on van den Broek's hypothesis that $N$ represents the number of units of positive electricity in the atomic nucleus. He did not show what this unit of positive electricity is nor how the nuclear charge may be computed.

Since the publication of Moseley's work there have been a number of attempts to apply Einstein's photoelectric equation, $\frac{1}{2}mv^2 = eV = vh$ to the energy of the cathode rays in a vacuum tube and the frequency of the $x$-rays which they excite. These attempts have generally been unsuccessful, apparently because the frequency of the $x$-rays has been measured by their absorption in aluminum. It is apparently for this reason that Barkla decides in his Bakerian Lecture of the present year that the quantum equation does not apply in the case of $x$-rays.

The recent work of Duane and Hunt,[2] Albert W. Hull[3] and David L. Webster[4] seems to show that this method of estimating the frequency of $x$-rays is not reliable, and that the Einstein equation does apply fully to the energy of cathode rays and the frequency of the $x$-rays which they excite, as well as to the "corpuscular rays" which are emitted by metals when excited by $x$-rays.

If we accept these conclusions, and if we make the further assumption that $x$-rays are due to electrons which are revolving in elliptical or circular orbits about a positively charged central nucleus with a rotational frequency equal to the frequency of the $x$-rays which they emit, we may compute the charge upon the central nucleus and show that it is directly proportional to the square root of the frequency of the $x$-rays, and hence

[1] Read before the Pacific Physical Society at Berkeley, California, Dec. 2, 1916.

[2] PHYS. REV., VI, 166 (Aug., 1915).

[3] Ibid., VII., 156 (Jan., 1916).

[4] Ibid., VII., 599 (June, 1916).

that it may also be computed from the Moseley equation by substituting the nuclear charge for the square root of the $x$-ray frequency.

Thus, from the Einstein equation, $mv^2 = 2h\nu$, and $\nu = mv^2/2h$. If the radiating electrons are moving in circular orbits with revolution frequencies which are equal to the vibration frequencies of the $x$-rays which they induce, we have for their orbital velocities $v = 2R\pi\nu$, where $R$ is the orbital radius and $\nu$ the revolution frequency of the electrons. Accordingly, $\nu = v/2R\pi$. Equating the above values of $\nu$, we have $mv^2/2h = v/2R\pi$ and

$$Rv = h/m\pi. \qquad \text{Equation 1.}$$

If we assign the ordinarily accepted values to $h$ and $m$, viz., $h = 6.55\ 10^{-27}$ and $m = 8.8\ 10^{-28}$, we have $Rv = 2.37$. If we substitute $\lambda$, the wave-length of the $x$-rays, for their frequency in the Einstein equation, we have $mv^2 = 2h \cdot 3 \cdot 10^{10}/\lambda$, and $v^2 = 6h \cdot 10^{10}/m\lambda$. Assigning the values already used to $h$ and $m$, we have

$$v^2 = 44.7 \cdot 10^{10}/\lambda \quad \text{and} \quad v = 6.69 \cdot 10^5/\sqrt{\lambda}. \qquad \text{Equation 2.}$$

If we assume that the revolving electrons are held in their orbits by the attraction of the electro-positive nucleus, we shall have the equation $mv^2/R = Qe/kR^2$, where $Q$ = the positive nuclear charge, $e$, the negative electronic charge and $k$ the specific inductive capacity within the atom. Since we know nothing of this specific inductive capacity, we can only neglect it in our equation, and we accordingly write $Q = mRv^2/e$.

From equation 1, $R = h/mv\pi$. Substituting this value, $Q = vh/e\pi$. calling $e = 4.77\ 10^{-10}$, $h/e\pi = 4.37\ 10^{-18}$. From Equation 2, $v = 6.69 \cdot 10^5/\sqrt{\lambda}$. Substituting these values in our equation for the value of $Q$, we have

$$Q = 2.95 \cdot 10^{-12}/\sqrt{\lambda}. \qquad \text{Equation 3.}$$

Since Moseley's equations give $\sqrt{\nu} = A(N - 1)$ for the $\alpha$ line of the K-radiation, while for the corresponding line of the L-radiation $\sqrt{\nu} = A(N - 7.4)$, it follows that the same equations, but with different values of $A$, must apply to the nuclear charges of the atoms which produce these spectral lines.

The nuclear charges for both the $\alpha$ and $\beta$ lines of the elements whose K-radiation wave-lengths have been measured by Moseley and others are given in Table I. as calculated from the equation $Q = 2.95 \cdot 10^{-12}/\sqrt{\lambda}$ and from the equation $Q = A(N - 1)$, where $A$ has a value of $8.51 \cdot 10^{-10}$ for the $\alpha$ line and $8.9 \cdot 10^{-10}$ for the $\beta$ line. The wave-lengths used in the computation are taken from Moseley's measurements as given on page 235 of Kaye's "*X-Rays*," from Malmer's paper in Phil. Mag., 28, 787 (1914) and from the abstract of a paper by Siegbahn and Stenström

in Science Abstracts, Vol. 19, No. 1129 (Oct. 25, 1916). The original paper in Phys. Zeits. from which the abstract was taken has not yet reached our library.

TABLE I.

*K—Radiation.*

| Atom. | No. | $\lambda\alpha$. | $\alpha$. | | $\lambda\beta$. | $\beta$. | |
|---|---|---|---|---|---|---|---|
| | | | $\frac{2.95.10^{-12}}{\sqrt{\lambda}}$. | $8.51(N-1)10^{-10}$. | | $\frac{2.95.10^{-12}}{\sqrt{\lambda}}$. | $8.9(N-1)10^{-10}$. |
| Na..... | 11 | 11.951 | 85.3 | 85.1 | | | |
| Mg..... | 12 | 9.915 | 93.7 | 93.6 | 9.477 | 96.5 | 97.9 |
| Al..... | 13 | 8.362 | 102 | 102 | 7.954 | 104.7 | 107 |
| Si..... | 14 | 7.136 | 110.3 | 110.6 | 6.744 | 113.6 | 115.7 |
| P..... | 15 | 6.168 | 119 | 119.2 | 5.808 | 122.5 | 124.7 |
| S..... | 16 | 5.360 | 127 | 127.6 | 5.018 | 132 | 133.5 |
| Cl..... | 17 | 4.730 | 135.6 | 136.1 | 4.394 | 140.5 | 142.4 |
| K..... | 19 | 3.747 | 155 | 153.3 | 3.456 | 159 | 160 |
| Ca..... | 20 | 3.363 | 160.6 | 161.5 | 3.090 | 167.5 | 169 |
| Se..... | 21 | 3.030 | 169 | 170 | 2.778 | 176.5 | 178 |
| Ti..... | 22 | 2.751 | 178 | 178.6 | 2.516 | 186 | 187 |
| V..... | 23 | 2.509 | 187 | 187 | 2.294 | 194.5 | 196 |
| Cr..... | 24 | 2.293 | 194.5 | 195.7 | 2.086 | 204 | 204.6 |
| Mn..... | 25 | 2.103 | 203 | 204 | 1.860 | 216 | 214 |
| Fe..... | 26 | 1.938 | 212 | 212.4 | 1.756 | 223 | 222.5 |
| Co..... | 27 | 1.798 | 220 | 221.6 | 1.629 | 230.3 | 231.5 |
| Ni..... | 28 | 1.662 | 228.4 | 229.8 | 1.506 | 239.7 | 240 |
| Cu..... | 29 | 1.549 | 238 | 238 | 1.402 | 250 | 249 |
| Zn..... | 30 | 1.445 | 244 | 246.6 | 1.306 | 258.5 | 258 |
| Y..... | 39 | 0.841 | 322 | 323 | 0.757 | 339 | 338 |
| Zr..... | 40 | 0.800 | 330 | 332 | 0.718 | 348.5 | 347 |
| Nb..... | 41 | 0.750 | 340.6 | 340.5 | | | |
| Mo..... | 42 | 0.716 | 349 | 349 | 0.631 | 372 | 367 |
| Ru..... | 44 | 0.641 | 368.5 | 366 | 0.569 | 392 | 383 |
| Pd..... | 46 | 0.587 | 387.5 | 383 | 0.522 | 408 | 400 |
| Ag..... | 47 | 0.562 | 393.5 | 392 | 0.499 | 418 | 410 |
| Cd..... | 48 | 0.535 | 403 | 400 | 0.471 | 430 | 418 |
| In..... | 49 | 0.511 | 412.5 | 409 | 0.448 | 441 | 427 |
| Sn..... | 50 | 0.485 | 424 | 417 | 0.427 | 452 | 436 |
| Sb..... | 51 | 0.449 | 441 | 426 | 0.396 | 468 | 445 |
| Ba..... | 56 | 0.383 | 476 | 468 | 0.334 | 511 | 490 |
| La..... | 57 | 0.373 | 483 | 477 | 0.326 | 517 | 498 |

The measurements of Moseley and of Siegbahn and Stenström agree with each other much more closely than either of them agrees with Malmer's measurements. Where more than one observer has measured the wave-length of the same line the means of the different measurements are taken in the table. All the wave-lengths for elements whose atomic numbers are above 47 are exclusively from Malmer's data.

In Table II., the same data are given for the principal line of the L-radiation as are given in Table I. for the two lines of the K-radiation. The wave-lengths used are taken from Moseley's tables and from a paper by Siegbahn and Friman in Phil. Mag., 31, 403 (1916). The values of $Q$ in column five are calculated from the equation $Q = 3.65(N - 7.4)\ 10^{-10}$.

TABLE II.

*L—Radiation.*

| Atom. | No. | λ. | $Q$. | |
|---|---|---|---|---|
| | | | $\frac{2.95 \cdot 10^{-12}}{\sqrt{\lambda}}$. | $3.65\ (N-7.4)\ 10^{-10}$. |
| Zr..... | 40 | 6.091 | 119.43 | 118 |
| Nb..... | 41 | 5.749 | 122.92 | 122.7 |
| Mo..... | 42 | 5.432 | 126.6 | 126.3 |
| Ru..... | 44 | 4.861 | 133.8 | 133.6 |
| Rh..... | 45 | 4.662 | 136.6 | 137 |
| Pd..... | 46 | 4.385 | 141.2 | 141 |
| Ag..... | 47 | 4.170 | 144.7 | 144.6 |
| Sn..... | 50 | 3.619 | 155 | 155.2 |
| Sb..... | 51 | 3.458 | 158.6 | 159 |
| La..... | 57 | 2.676 | 180 | 181 |
| Ce..... | 58 | 2.567 | 184.5 | 184.5 |
| Pr..... | 59 | 2.471 | 188 | 188 |
| Nd..... | 60 | 2.382 | 191.4 | 192 |
| Sa..... | 62 | 2.208 | 199 | 199 |
| Eu..... | 63 | 2.130 | 202 | 203 |
| Gd..... | 64 | 2.057 | 203.5 | 206 |
| Ho..... | 66 | 1.914 | 214 | 214 |
| Er..... | 68 | 1.790 | 220 | 221 |
| Ta..... | 73 | 1.525 | 238.7 | 239 |
| W..... | 74 | 1.486 | 242 | 242.5 |
| Os..... | 76 | 1.397 | 250 | 250 |
| Ir..... | 77 | 1.354 | 254 | 254 |
| Pt..... | 78 | 1.316 | 256 | 257 |
| Au.. ..... | 79 | 1.287 | 258.5 | 261 |
| | From the data of Siegbahn and Friman. | | | |
| Au..... | 79 | 1.280 | 261 | 261 |
| Hg..... | 80 | 1.243 | 267 | 265 |
| Tl..... | 81 | 1.209 | 268 | 268.4 |
| Pb..... | 82 | 1.179 | 272 | 272 |
| Bi..... | 83 | 1.144 | 276 | 276 |
| Th..... | 90 | 0.971 | 302 | 301 |
| U..... | 92 | 0.919 | 308 | 308.4 |

The relation of the nuclear charges to the atomic numbers which are shown in Tables I. and II. are shown graphically for the K $\alpha$ line in Fig. 1 and for the principal line in the L-radiation in Fig. 2.

It will be seen from the tables, as well as from the formulæ from which

they are computed, that the positive charges within the orbits of the electrons which give the L-radiation are less than those inside the orbits of the electrons which emit the K-radiation in the same atom.

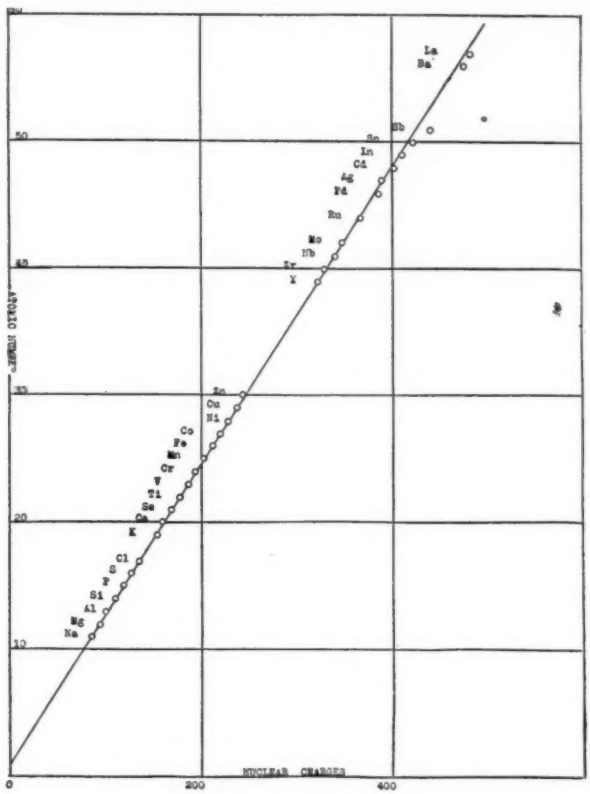



Fig. 1.

This can be explained on the assumption that the electrons which give the K-radiation, and presumably other electrons, lie within the orbits of those which emit the L-radiation. If this be the correct explanation, the difference in the nuclear charges computed from the two kinds of radiation for the same element should be some integral multiple of the electronic charge.

There are nine elements for which both the K and L wave-lengths are given in the table. When the differences of the computed charges for these nine elements are taken, they are found to be in every case very nearly integral multiples of the electronic charge, so that $Q_K - Q_L = e(N + 5)$, where $e$ has a mean value for the nine elements of $4.75\ 10^{-10}$ E.S.U. This relation is shown in Table III.

The orbital radii of the revolving electrons whose wave-lengths are

TABLE III.

| Atom. | No. | $Q_K$. | $Q_L$. | $Q_K - Q_L$. | $\frac{Q_K - Q_L}{(N+5)}$. |
|---|---|---|---|---|---|
| Zr..... | 40 | 332 | 118 | 214 | 4.76 |
| Nb..... | 41 | 340.5 | 122.7 | 217.8 | 4.74 |
| Mo..... | 42 | 349 | 126.3 | 222.7 | 4.74 |
| Ru..... | 44 | 366 | 133.6 | 232.4 | 4.74 |
| Pd..... | 46 | 383 | 141 | 242 | 4.75 |
| Ag..... | 47 | 392 | 144.6 | 247.4 | 4.76 |
| Sn..... | 50 | 417 | 155.2 | 261.8 | 4.76 |
| Sb..... | 51 | 426 | 159 | 267 | 4.77 |
| La..... | 57 | 477 | 181 | 296 | 4.77 |

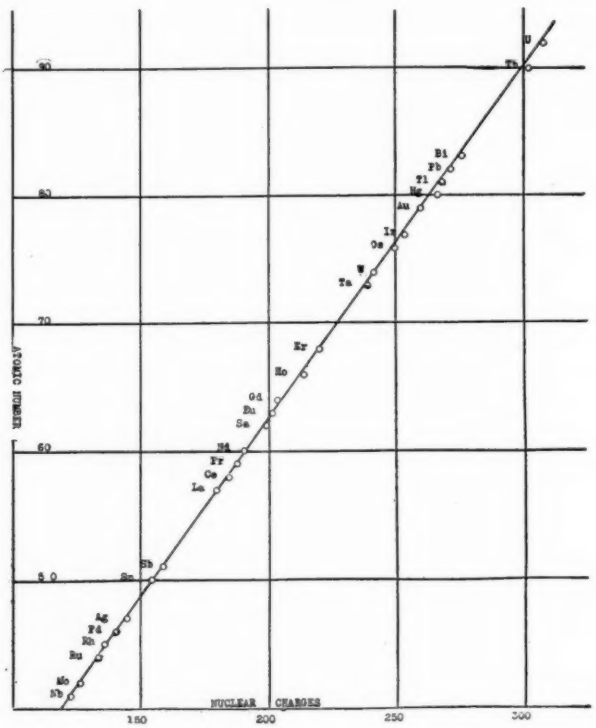



Fig. 2.

known may be calculated from the equation $R = 3.542\ \lambda 10^{-6}$. They vary in the case of the K-radiation from $12.25\ 10^{-10}$ for sodium to $2.17\ 10^{-10}$ for lanthanum. It has been shown in a previous paper by the present writer[1] that the atomic radii calculated in this way from the convergence wave-lengths in spectral series give values of the same order of magnitude and of the same relative size as when calculated by other methods.

[1] Astrophysical Journal (Nov., 1916).

It may also be seen that the orbital radii vary inversely as the nuclear charges. If this decrease of the effective positive charge within the electronic orbits with the increase of the orbital radii is due to the number of electrons surrounding the positive nucleus, it would seem to follow that these electrons must be spaced at equal intervals outwards from the center.

In a paper on "The Specific Inductive Capacity of Certain Metals,"[1] the present writer has called attention to the fact that in order that the Einstein equation may apply to a system of electrons revolving in circular orbits at different distances from the center they must be held by centripetal forces which vary as the inverse cubes of their respective distances from this center. It was suggested that this result might be accomplished if the specific inductive capacity of the system increased proportionally as the distance from the center of the system. It may also be accomplished if the negative charges within the electronic orbit increase directly as the radius of the orbit.

Since the $\beta$ lines of the K-radiation are of shorter wave-length than the $\alpha$ lines, it would seem that they must be used in computing the true nuclear charges of the atoms. If these electrons are really the closest electrons to the central nucleus, and if the nuclear charge may be calculated from the Moseley equation, using $Q$ instead of $\sqrt{\nu}$, then the nuclear charges of all the atoms of higher atomic weight than helium must be integral multiples of the charge on the helium nucleus, and presumably of the charge of an $\alpha$-particle. It has generally been assumed that the charge of an $\alpha$-particle has exactly twice the magnitude of the electronic charge, viz. $9.54 \cdot 10^{-10}$ E.S.U. The mean values from the determinations of Rutherford and Geiger and of Regener give $9.4 \cdot 10^{-10}$ E.S.U. Since the nuclear charge which is the constant factor determined from the $\beta$ lines of the K-radiation is only $8.9 \cdot 10^{-10}$ E.S.U., it is about 5.3 per cent. less than the values given by the above measurements.

If, as some believe, there is a radiation of still shorter wave-length than the K-radiation, its wave-lengths should be used in computing the true nuclear charge, and these would give a larger value for the nuclear charge of helium than do the wave-lengths of the K-radiation. The J-radiation, if it exists, should be looked for in elements of low atomic weight and with cathode rays of higher speed than those required to excite the K-radiation. If the value of $A$ used in computing the nuclear charge from Moseley's equation for the J-radiation should prove to be the estimated value of the charge on the $\alpha$-particle, a voltage of 1325 should be required to excite the J-radiation in sodium, and the wave-length of the J-radiation for sodium should be $1.45 \cdot 10^{-8}$.

[1] PHYS. REV., VIII., 89 (July, 1916).

Since the radioactive elements in their transformations give off $\alpha$-particles and electrons and form new elements, the temptation is great to assume that all elements are built up from some one or two small nuclei by the addition of $\alpha$-particles and electrons. In that event, leaving the electrons entirely out of consideration, the nuclear charges should increase with the atomic weight only about half as fast as they seem to increase. Thus the atomic weight should increase by four units while the nuclear charge increases by $8.9 \cdot 10^{-10}$ E.S.U.

Notwithstanding this difficulty, and whether a J-radiation having a higher nuclear charge than the K-radiation be found, or not, the important relation which Moseley has shown between the frequency of the $x$-rays and the atomic numbers of the respective atoms are explained in a very simple manner if the radiating electrons are moving in circular orbits about the positive atomic nucleus. The further fact that in the "corpuscular radiation," induced when $x$-rays are stopped by metals, electrons are driven off with velocities equal to the orbital velocities of the electrons which induce the exciting $x$-rays seems to suggest the possibility that $x$-rays are really produced by electrons which are captured for short periods by the positive nuclei when cathode rays are shot through the atoms.

That charges of similar magnitude to those computed here are actually associated with the positive sub-atoms may be shown from other considerations. Attention has been called by the present writer in a number of papers to what he has called the characteristic ionic charges in electrolysis. These charges are calculated on the assumption that the ions which carry charges in electrolysis are free for very short intervals of time, and that their movement in the solution is accelerated throughout their entire free path by the electromotive force which causes the current. This being true, the acceleration of an ion would be proportional to its charge and inversely proportional to its mass. If the ionic mobilities are proportional to these accelerations, which will be the case if the ions of different elements are free for the same time periods when in solutions of the same molecular concentration, the ionic charges will be proportional to the products of the ionic mobilities into the ionic masses.

Assuming the charge on the hydrogen ion to be $5 \cdot 10^{-10}$ E.S.U., charges have been computed for all the ions whose electrolytic mobilities are known. For the alkali metals and the chlorine group of elements these charges are as follows:

When the ionic charges calculated in this way are compared with the nuclear charges calculated from $x$-ray wave-lengths, they are found to be closely proportional to them within each group of similar elements.

| Ion. | Charge $\times 10^{10}$. | Ion. | Charge $\times 10^{10}$. |
|---|---|---|---|
| Na..... | 15.7 | F..... | 14.6 |
| K..... | 39.6 | Cl..... | 36.5 |
| Rb..... | 91.2 | Br..... | 84.9 |
| Cs..... | 142 | I..... | 132.5 |

Thus the nuclear charges for the L-radiation as calculated from the formula $Q = 3.65(N - 7.4)$ may also be calculated for Na, K, Rb and Cs from the equation $Q = 1.22(E - 5)$, where $Q$ represents the nuclear charges and $E$ the ionic charges in electrolysis. For the elements F, Cl, Br and I, the formula is $Q = 1.362\ (E - 10.5)$. These relations are shown graphically in Figs. 3 and 4. A similar relation exists in the case

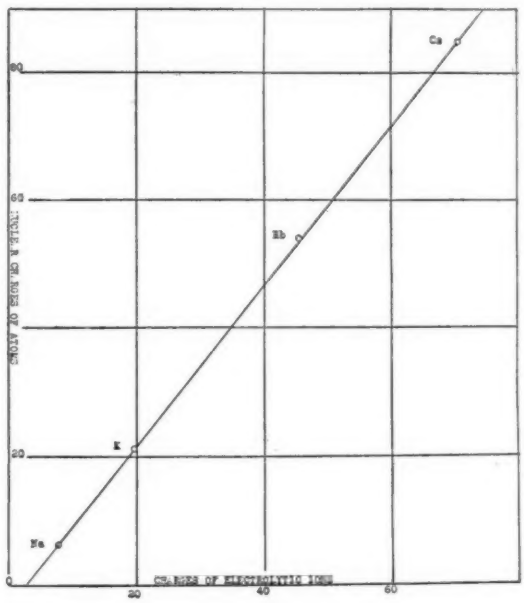



Fig. 3.

of the elements Mg, Ca, Sr and Ba. Thus the ionic mobility of these elements may be calculated from the wave-lengths of their $x$-ray spectrum as accurately as from electrolysis.

The charges on the positive ions given off by hot metals have been calculated from Richardson and Hulbirt's values of $e/m$ given in Phil. Mag., XX., 545 (1910), and by other methods. These values for the charges of Ni, Pd and Pt have been taken as follows: Ni, 51.4; Pd, 88.9; Pt, 157. They may be used to calculate the nuclear charges in the L-radiation from the formula $Q = 1.7(E - 5)$ to within one per cent. of the above values. This relation is shown graphically in Fig. 5.

Since practically all the properties of the elements which depend upon cohesion have been shown to be closely related to the ionic charges cal-

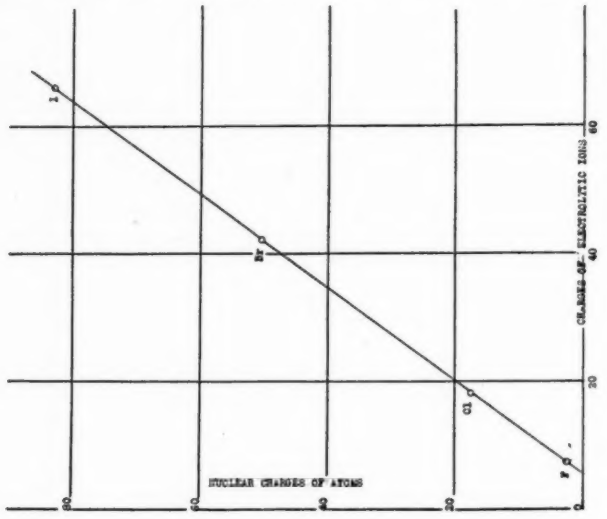



Fig. 4.

culated from electrolysis, and since it is now shown that the characteristic $x$-rays given off by the atoms may also have their wave-lengths computed from the mobilities of their ions in electrolysis, it seems to be definitely established that these charges are the most fundamental characteristics of the elements which have yet been observed. Conversely, the evidence of the relation between the ionic charges and the nuclear charges calculated on the assumption that the $x$-radiation is produced by electrons revolving in orbits about a central positive nucleus seems to make this assumption very probable.

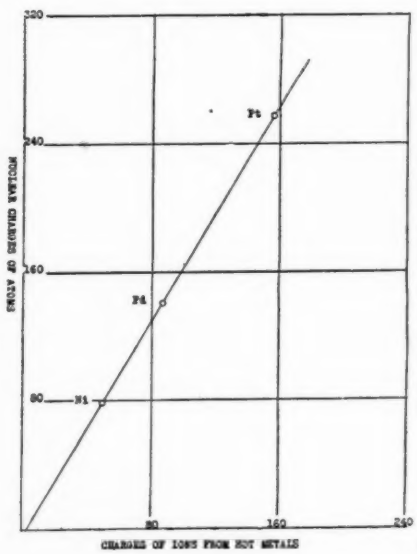



Fig. 5.

## SUMMARY.

Assuming, as seems very definitely established, that the Einstein photoelectric equation $Ve = h\nu$, applies to characteristic $x$-rays and their exciting cathode rays, it is shown that if the electrons which emit the

$x$-radiation are revolving in orbits about a central positive nucleus, the charge of this nucleus and the orbital radius of the revolving electron may be calculated from equations of the same form as Moseley's equations for the X-ray frequency.

The nuclear charges, when computed, are found to be integral multiples of a primary charge, which in the case of the K-radiation is the nuclear charge of helium, and which for the $\beta$ lines of the K-radiation has a value of $8.9 \cdot 10^{-10}$ E.S.U.

The van den Broek hypothesis is sustained to the extent that the nuclear charge is found to increase with the atomic number.

It has previously been shown that the orbital radius calculated in this manner from the convergence wave-lengths in known spectral series agrees very closely with the atomic radius calculated from other considerations.

In the case of the nine elements for which both the K and L characteristic radiations are known, it is shown that the nuclear charges calculated from the two classes of radiation differ in every case by an integral number of electronic charges, so that this difference may be calculated from the equations $Q_K - Q_L = e(N + 5)$.

It is shown that the nuclear charges calculated from $x$-ray wave-lengths may also be calculated, within groups of similar elements, from the ionic charges calculated from the mobility of the ions in electrolysis and from the charges of positive ions given off by hot metals. It is thus possible to calculate the ionic mobilities from the $x$-ray wave-lengths, and *vice versa*.

STANFORD UNIVERSITY,
November 25, 1916.

# THE MAGNETIC PROPERTIES OF IRON, NICKEL AND COBALT ABOVE THE CURIE POINT, AND KEESON'S QUANTUM THEORY OF MAGNETISM.

BY EARLE M. TERRY.

IN 1905, Langevin[1] proposed a kinetic theory of magnetism by means of which he was able to account in a remarkable way for many of the thermo-magnetic phenomena of both para- and dia-magnetic bodies. Assuming that the molecules of a paramagnetic substance are, as in a gas, free to orient themselves in all directions and that, with respect to an external field, the distribution of directions of their magnetic axes under the influence of thermal agitation follows the Maxwell-Boltzmann law, he derived, by purely statistical methods, the law previously obtained experimentally by Curie,[2] that the susceptibility is inversely proportional to the absolute temperature.

Introducing the idea of an internal magnetic field proportional to the intensity of magnetization, Weiss[3] has extended the method of Langevin to ferro-magnetic substances, and obtained a relation between intensity and temperature, which, when put in the form of corresponding states, represents the experimental facts with a fair degree of accuracy. He has further shown that in the region of "induced ferro-magnetism," *i. e.*, in the temperature interval immediately above the magnetic transformation point, called the "Curie Point," the susceptibility should follow a modified form of Curie's Law, namely, inversely proportional to the excess of temperature above the transformation point where the constant of proportionality is the same as would have held for the substance, if, by suppression of the mutual actions between molecules, it had remained paramagnetic.

As a result of their study of magnetic properties at very low temperatures, Weiss and Kammerlingh Onnes[4] found that the saturation values of the magnetic moments per gram atom for Fe, Ni, and Co are to one another in simple ratios; namely, 11 : 3 : 9. To the aliquot part common to the magnetic moment of the gram atoms, Weiss has given

[1] Langevin, Ann. de chem. et phys. (8), 5, p. 70, 1905.

[2] Curie, Oeuvres, p. 255, 1908.

[3] Weiss, Arch. des Sci. phys. et nat. (4), Vol. 31, p. 5, 1911.

[4] Weiss and Kammerlingh Onnes, Comm. phys. Lab. Leyden, No. 114, p. 3, 1910.

the name of gram-magneton and this quantity, divided by the Avogadro number, gives the moment of the ultimate magnetic corpuscle which he has called the magneton. This he supposes, for magnetism, to play a rôle analogous to the electron for electricity. His theory further shows that the number of magnetons per molecule is related to the Curie constant in a simple manner, so that it is easy to test the existence of the magneton by determining whether or not the number per molecule is an integer. Using the results of Pascal,[1] Weiss and Foëx,[2] Leibknecht and Wills,[3] Miss E. Feytis[4] and others, he found that the ferromagnetic substances at high temperatures and their salts at ordinary temperatures give, for the number of magnetons per molecule, numbers which rarely depart from integers by more than one or two per cent. Similar results were obtained for a large number of alloys of iron, nickel and cobalt.

On the other hand, Honda[5] and his co-workers, after examining a large number of substances, both elements and compounds, have amassed a large amount of data showing that Curie's law is not a general law of nature and that the existence of the magneton, at least as based on the simple theory of Weiss, is open to serious question.

In the cryogenic laboratory at Leyden, Kammerlingh Onnes[6] and his co-workers have extended their thermo-magnetic studies to very low temperatures with a view to testing the validity of Curie's law in this region and have found that, as regards their departures therefrom, substances may be divided into three general groups:

1. The relation, $\chi T$ = constant, holds down to the temperatures of liquid nitrogen, but below this, $\chi T$ continually decreases.

2. The product $\chi T$ diminishes at once starting from room temperature. The relation $(T + \Delta)$ = constant holds with fair accuracy as far as liquid nitrogen temperatures but below this, unsystematic deviations appear.

3. Susceptibility changes very little with temperature and, in the interval between liquid nitrogen, and liquid hydrogen, is almost independent of temperature.

Kunz[7] has shown that the elementary magnet in the Weiss-Langevin theory is not the magneton mentioned above but is a much larger aggregate consisting of a definite number of atoms: *e. g.*, Fe, 2; and Ni, 6.

[1] Pascal, Ann. de Chim. et Phys., Vol. XVI., p. 531, 1909.
[2] Weiss and Foëx, Archiv. des Sci. phys. et nat., Vol. XXXI.. 1911, p. 4 and 89.
[3] Liebknecht and Wills, Ann. des Phys., I., p. 178, 1900.
[4] Miss E. Feytis, Comptes rendus, Vol. CLII., p. 708; 1911.
[5] Honda, Science Reports Tohoku Univ., Vol. 4, p. 215, 1915.
[6] K. Onnes, Perrier, Oosterhuis, Leyden Comm. No. 5, 116, 122*a*, 129*b*, 132*e*, 134*d*.
[7] Kunz, PHYS. REV., Vol. XXX., p. 359; 1910.

Stifler[1] has studied cobalt and found, for the atoms per elementary magnet, four.

The classical work of Curie was carried out at a time when the subject of pyrometry was in its infancy and his temperature scale is in error by at least twenty degrees. Weiss and Foëx could not extend their work to melting point temperatures of the ferro-magnetic substances, because oxidation and errors from the effects of the hydrogen atmosphere which filled their furnace chamber make their results open to question. The same criticisms may be made of the word of Honda whose experimental equipment was quite similar except that he used an atmosphere of nitrogen. Moreover, in the work of Weiss and Foëx, certain deviations from Curie's law are apparent in the case of iron, nickel, and cobalt. Since it is of extreme importance, from the standpoint of the magneton theory, to have accurate information regarding the thermo-magnetic behavior of these cardinal magnetic substances, the present investigation was undertaken with these objects in view:

1. To extend the work up to the melting point.
2. To eliminate errors due to effects of oxidation by working in a good vacuum.
3. To eliminate the effects of occluded gases by melting the specimens in a vacuum before testing.
4. To improve previous temperature measurements.

*Method of Measurement.*—The method which seemed best adapted for the purpose is that of Faraday in which the magnetic susceptibility is deduced from the mechanical force exerted upon a specimen in a non-uniform field. For an object of small dimensions placed in the plane of symmetry between the poles of a magnet, the force $F$ in a direction $X$ at right angles to the field $H$ is given by the expression

$$F = m\chi H \frac{dH}{dx}, \qquad (1)$$

where $m$ is the mass of the specimen, and $\chi$ its susceptibility referred to unit mass. If this is to be an absolute method, it is obviously necessary to determine both the field strength and its gradient over the volume occupied by the specimen. However, if a body of known susceptibility is available, the product $H(dH/dx)$ may be determined for a particular place in the field by substituting it for the test specimen and measuring the force upon it. This method was followed in the present work.

The force $F$ was measured by means of a modified form of the Curie balance as shown in Fig. 1. The specimen $m$ was supported at the desired

[1] Stifler, PHYS. REV., Vol. XXXIII., p. 268; 1911.

point in the field by means of a slender porcelain tube carried by an aluminum arm and counterpoised by an adjustable threaded lead nut *W*. When the magnet *NS* is energized, the specimen is urged toward the stronger part of the field and the couple due to this force is counter-

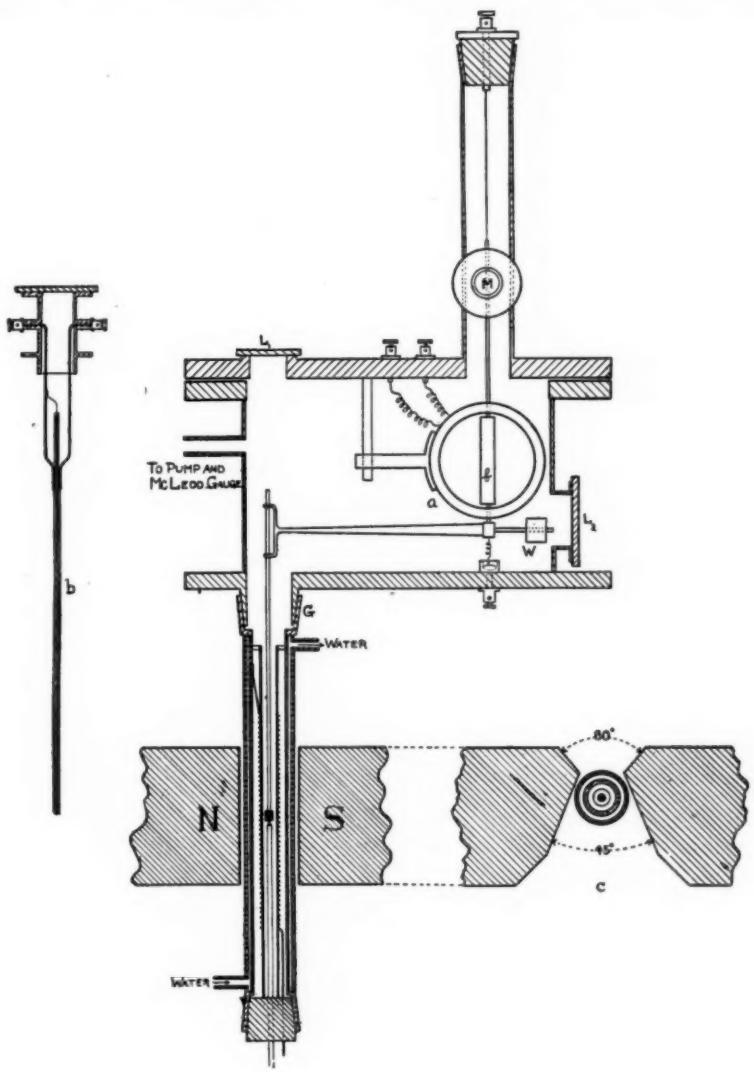



Fig. 1.

balanced by the electrodynamic action between the currents supplied to the coils *a* and *b*, the former of which is fixed while the latter is mounted on the moving system. A spot of light focused by the concave mirror *M* on a distant scale indicates when a balance has been obtained. The

substitution of the coils and optical lever for the torsion fiber and divided head constitutes the principle modification of the original Curie balance. Since the torque is proportional to the current in each coil, this form of instrument may be used for large ranges of forces and has the advantages of high sensitivity and easy control when mounted within a vacuum chamber. Current is led to the moving coil by means of the suspending phosphor bronze ribbon, and away by a small copper wire dipping into a mercury cup below.

*Experimental Details.*—The furnace consisted of a tube of Berlin porcelain 8 mm. internal diameter and 20 cm. in length closely wound over a distance of 15 cm. with a .5 mm. tungsten wire. Outside of this tube there was placed another porcelain tube, not shown in the figure, and the whole mounted inside a water-cooled jacket. This container was supported from the bottom of the vacuum chamber by means of the ground joint *G*. An input of 750 watts sufficed for a temperature of 1500° C.

Temperature was measured by means of an Heraeus platinum platinum-rhodium thermo couple supported by a porcelain tube mounted upon the plug which closed the lower end of the furnace chamber. The junction was held immediately below the specimen. In its calibration, the following melting points were used: Cadmium, 320.9° C.; antimony, 630° C.; gold, 1062° C.; and nickel[1] 1452° C. In using the first two of these, the test specimen was swung to one side of the furnace and a tiny crucible of pure magnesia containing a small particle of the melt specimen was lowered through the window $L_1$, till it hung beside the test specimen. A vacuum was then obtained and the temperature gradually raised. The specimen was illuminated by light from a nearby arc reflected into the furnace by means of a prism placed upon $L_1$ and observed by a telemicroscope. The gold and nickel points were obtained by means of the apparatus shown in *b*, Fig. 1. The melt specimen, in the form of a narrow strip of foil, was supported by platinum wires at the end of a slender porcelain tube inserted through $L_1$ and made tight by means of a ground joint. The foil was used as a fuse in an electric circuit containing a relay which actuated a buzzer when the melt occurred. It is to be observed that the thermocouple was thus calibrated in the same furnace and under exactly the same conditions as when used for the measurement of temperature.

In the early part of the work, a tungsten-iron couple was used with good results. By properly adjusting the current and voltage, it is possible by forming an arc between them, to weld tungsten and iron wires in the

[1] Burgess and Waltenberg, Bulletin Bureau Standards, Vol. 10, p. 79, 1914, have shown that nickel possesses a very sharp melting point and, for work in a vacuum, it furnishes a very satisfactory standard temperature.

air. Such a thermo element has an inversion point near 300° C. and gives a thermoelectric power about twice that of platinum platinum-rhodium. It is satisfactory for work in a vacuum or hydrogen.

The magnet employed was a powerful one weighing 800 lbs. and required 4 K. W. for full excitation. It had pole pieces 10 cm. in diameter and was mounted upon a carriage which rested upon two rows of steel balls running in grooves. It could thus be easily moved to and from the test specimen by means of a slow motion screw operated from the observation table two meters distant. Since the pull upon the specimen depends upon its location in the field, it is necessary to work with it always at the same position. The point selected for this purpose was the one for which the pull is greatest, since it is easy to find and gives maximum sensitivity. As the force is proportional to both the magnitude and gradient of the field, it is desirable to have their product constant over as large an interval as possible to minimize the error due to variations in location of the specimen. By giving the pole faces the peculiar shape shown in *C*, Fig. 1, it is possible to obtain an $X$–$F$ curve with a very broad peak. A careful study of the particular field used in this work, carried out by means of a small travelling flip coil, showed that the slope of the $H^2$–$X$ curve varied by less than one per cent. over a distance of 1 cm. in the neighborhood of the position of maximum pull. Since the specimens used never exceeded 4 mm. in diameter, the error due to faulty location is very small. The method followed in each case was to energize the magnet at the normal working field, usually 9,000 gauss, and measure the force for a series of magnet positions on each side of the position desired and from the curve determine the point of maximum $F$.

The vacuum was obtained by a molecular pump and read on a McLeod gauge. Since the porcelain furnace tubes occlude large quantities of air and water vapor, it was necessary to bake them out at low temperatures for a long time, *e. g.*, several days. By taking proper precautions in this respect, it was possible to operate at the melting point of iron with a pressure which seldom exceeded .0005 mm. of Hg. The pump was run continuously during all measurements. Two dishes containing $P_2O_5$ were placed within the vacuum chamber near the opening to the furnace —a precaution absolutely essential to the life of a tungsten furnace and the constancy of a platinum thermo-couple used within it.

Current was supplied to both the magnet and furnace from storage batteries and controlled by water-cooled rheostats, which made possible the maintenance of very constant fields and temperatures.

*Standardization of the Curie Balance.*—As standard substances, four solutions, nickel nitrate, nickel sulphate, cobalt nitrate, and cobalt

sulphate, were used, whose susceptibilities were determined by the Quincke absolute method. For this purpose, the magnet was placed with its yoke in a vertical position and pole tips of 4 cm. diameter attached with a gap of 6 mm. The field strength was measured by three small independent flip coils with different numbers of turns and different diameters wound on carefully turned vulcanite spools. Their diameters were measured by means of a travelling microscope. The ballistic galvanometer was standardized in terms of a mutual inductance wound on a marble spool and certified by the U. S. Bureau of Standards. The change in level of the solution was read by a microscope travelling vertically with the apparatus so arranged that the capillary tube in which the liquid under test was placed travelled with it, which enabled the readings to be taken always through the same spot in the tube to eliminate errors due to unequal capillarity. A jacket through which water of constant temperature circulated surrounded the capillary tube and maintained the temperature of the solution under test invariable. The outstanding discrepancies in the constant of the Curie balance, as determined by these four different solutions, never exceeded one half of one per cent.

*The Test Specimens.*—The iron used in this investigation was Burgess electrolytic and was part of the same deposit used in a previous investigation[1] and is known to have a purity, as regards other metals, of 99.98 per cent. The nickel and cobalt were obtained from Kahlbaum in the form of powders guaranteed to be cobalt and nickel free respectively. All specimens were freed from occluded gases by melting in vacuo in crucibles cut from plates of pure magnesia from the Berlin Porcelain Factory. This is believed to be a very important precaution, as a large evolution of gas at the melting point is evidenced by a vigorous boiling which lasts for several seconds and then subsides. In fact, experience has shown that it is quite impossible to obtain consistent results at high temperatures unless gases are thus eliminated. This has also been noted by Burgess[2] in his work on the temperatures of the $Ar_2$ and $Ar_3$ points for iron. Several samples of electrolytic iron melted in vacuo prepared by Yensen[3] were also tested.

*Results.*—The results for beta iron are given in Table I. and shown graphically in Fig. 2.

It is to be noted that iron in this region obeys Curie's law only approximately, for, when the reciprocal of susceptibility is plotted against

[1] Terry, PHYS. REV., Vol. XXX., p. 133.
[2] Burgess, Bull. Bur. Stds., Vol. 10, p. 315, 1914.
[3] Yensen, Bulletin, Univ. of Ill., No. 72.

temperature, instead of a straight line, a curve with a marked upward concavity is obtained.

TABLE I.

Beta iron. Susceptibility as a function of temperature.

| Temp. | $\chi \cdot 10^{-6}$. | $\frac{1}{\chi} \cdot 10^{4}$. | Temp. | $\chi \cdot 10^{-6}$. | $\frac{1}{\chi} \cdot 10^{4}$. |
|---|---|---|---|---|---|
| 796 | 1,514 | .0661 | 860 | 379.0 | .2638 |
| 806 | 1,111 | .0900 | 874 | 327.2 | .3056 |
| 815 | 862.5 | .1160 | 878 | 297.6 | .3360 |
| 826 | 701 | .1426 | 889 | 267.5 | .3739 |
| 840 | 532 | .1879 | 896 | 241.4 | .4141 |
| 849 | 456 | .2193 | 907 | 205.4 | .4861 |
| 854 | 427 | .2342 | 916 | 180.7 | .5534 |

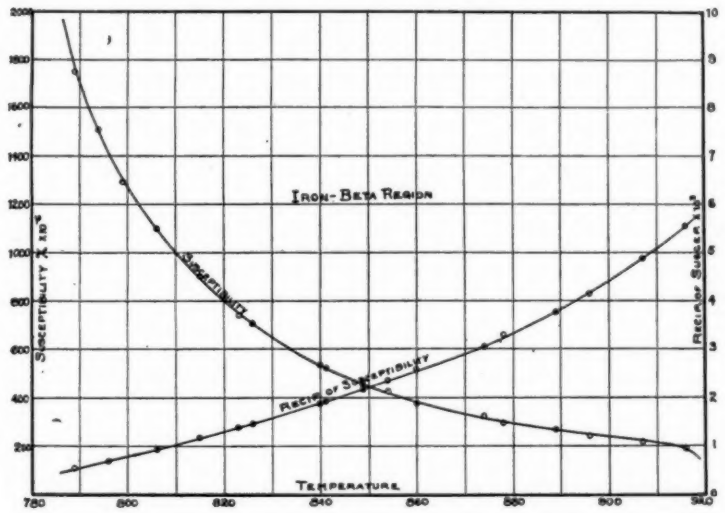



Fig. 2.

*Electrolytic Iron Melted in Vacuo, Beta Region.*

The results for iron in the gamma and delta regions are given in Table II. and plotted in Fig. 3.

It is to be noted that in the gamma region, Curie's law is not even approximately obeyed. We have here an interval of nearly five hundred degrees throughout which the susceptibility remains almost constant. At 1407° C. there is an abrupt increase when the transformation to the delta form occurs. Throughout the delta region, which extends to the melting point (1530° C.), the susceptibility decreases approximately linearly with the temperature. Electrolytic iron, as deposited, shows slightly greater values of susceptibility in the gamma and smaller in the delta regions than the same iron after being melted in vacuo. The

melting point is less sharply marked also, for fusion apparently begins at 1500° C. and extends over an interval of about twenty-five degrees. These specimens consistently clung to the supporting porcelain rod after melting so that observations on molten iron were obtained over a small interval. The susceptibility shows a marked decrease on melting and

TABLE II.

*Iron. Gamma and Delta Regions.*

| Temp. | $\chi \cdot 10^{-6}$. | Temp. | $\chi \cdot 10^{-6}$. | Temp. | $\chi \cdot 10^{-6}$. |
|---|---|---|---|---|---|
| 929 | 26.95 | 1197 | 25.54 | 1409 | 37.60 |
| 961 | 26.81 | 1224 | 25.37 | 1412 | 37.96 |
| 991 | 26.71 | 1245 | 25.19 | 1417 | 37.82 |
| 1026 | 26.40 | 1285 | 24.98 | 1425 | 37.69 |
| 1056 | 26.38 | 1343 | 24.58 | 1432 | 37.20 |
| 1071 | 26.32 | 1359 | 24.55 | 1442 | 36.72 |
| 1086 | 26.20 | 1370 | 24.39 | 1464 | 35.44 |
| 1109 | 25.96 | 1374 | 24.30 | 1483 | 34.38 |
| 1125 | 25.93 | 1400 | 24.40 | 1502 | 33.04 |
| 1145 | 25.87 | 1400 | 24.83 | 1515 | 31.93 |
| 1151 | 25.85 | 1400 | 29.87 | 1532 | 30.49 |
| 1164 | 25.76 | 1403 | 35.58 | | |

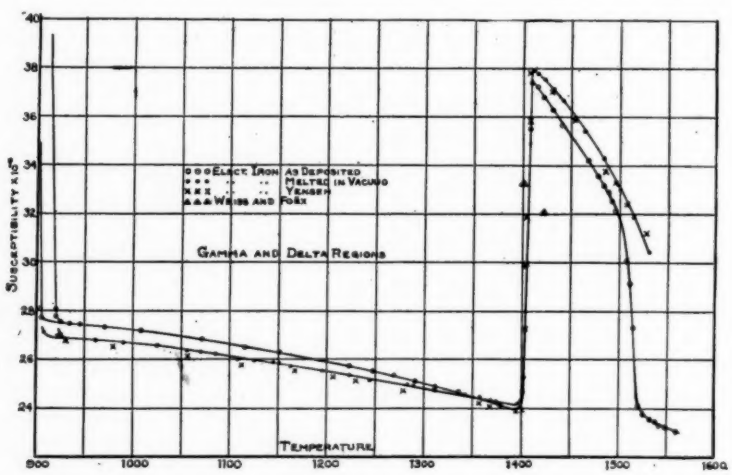



Fig. 3.

Gamma and delta iron. Susceptibility as a function of temperature.

has values approximately corresponding to an extension of the curve for the gamma region. Iron which had previously been melted in vacuo showed a sharp melting point and always dropped from the support.

A special study was made of the transformation from the beta to gamma and gamma to delta regions, the results of which are shown in Fig. 4.

These were carried out by causing the temperature to change very slowly while susceptibility changes were followed closely. As water cooled rheostats were used and the furnace and magnet currents were supplied by storage batteries, it was possible to control conditions very accurately. The rate of change of temperature seldom exceeded 10

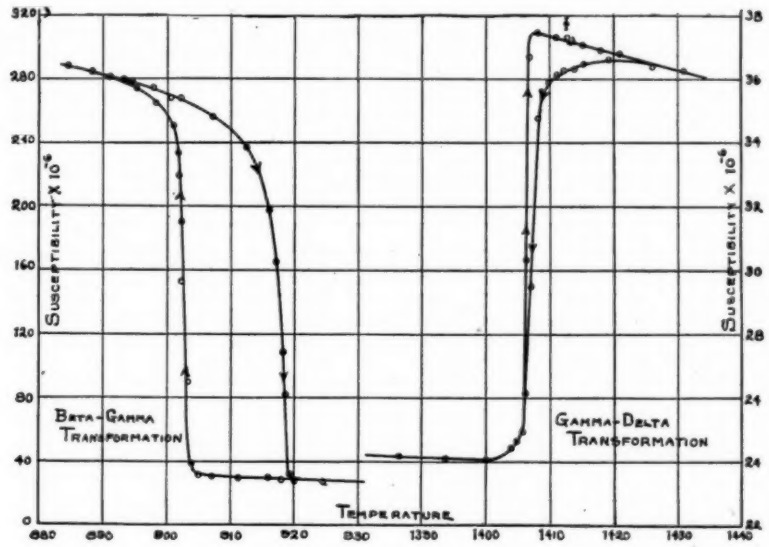



Fig. 4.

Beta-gamma and gamma-delta transformation in iron.

degrees per half hour in the neighborhood of a transformation point. To insure accurate temperature readings, the thermo couple was calibrated during each run. For example, at the beta-gamma transformation, a gold melt specimen was introduced beside the test specimen as explained above. The susceptibility curve for rising temperature was then obtained and extended up to the melting point of gold (1062° C.), after which the curve for decreasing temperature was taken. For the gamma-delta transformation, the thermo couple was checked at the nickel point in the same manner.

It is to be observed that these transformations are of entirely different characters magnetically. For the former, there is a marked hysteresis. The steepest part of the curve occurs at 918° C. on rising and at 903° C. on falling temperatures. For the latter, no real hysteresis can be detected. The steep part of the curve on falling coincides with that on rising within the limits of experimental accuracy. A marked peculiarity, however, is the fact that on cooling the return to the gamma state is not abrupt but begins rather gradually. This peculiarity was always found,

even though the specimen had been heated to within a few degrees of the melting point. The results given here for the beta-gamma transformation correspond qualitatively to those obtained by Burgess[1] for the heat changes. He gives for the temperature at which maximum rates of absorption and evolution occur 898° C. and 909° C. respectively.

The results for nickel are given in Table III. and plotted in Fig. 5. Of the three metals studied, nickel alone shows a tendency to obey Curie's law, but for limited regions only. Starting at the transformation point,

TABLE III.

| Temp. | $\chi \cdot 10^{-6}$. | $\frac{1}{\chi} \cdot 10^{6}$. | Temp. | $\chi \cdot 10^{-6}$. | $\frac{1}{\chi} \cdot 10^{6}$. |
|---|---|---|---|---|---|
| 381 | 820. | .1219 | 782 | 13.06 | 7.650 |
| 383 | 723. | .1383 | 807 | 12.53 | 7.980 |
| 385 | 678. | .1478 | 820 | 11.72 | 8.530 |
| 387 | 611. | .1638 | 849 | 11.12 | 9.000 |
| 391 | 498. | .2008 | 866 | 10.61 | 9.425 |
| 393 | 377. | .2653 | 890 | 10.13 | 9.870 |
| 403 | 283. | .3530 | 924 | 9.495 | 10.53 |
| 411 | 219. | .4570 | 961 | 8.845 | 11.31 |
| 416 | 183.7 | .5460 | 1010 | 8.214 | 12.18 |
| 420 | 177.2 | .5645 | 1033 | 7.842 | 12.75 |
| 423 | 149.2 | .6700 | 1051 | 7.734 | 12.93 |
| 435 | 126.7 | .7890 | 1074 | 7.550 | 13.24 |
| 445 | 102.3 | .9800 | 1099 | 7.318 | 13.66 |
| 467 | 74.90 | 1.334 | 1107 | 7.275 | 13.74 |
| 491.5 | 56.85 | 1.760 | 1147 | 6.968 | 14.35 |
| 508 | 47.90 | 2.046 | 1172 | 6.774 | 14.76 |
| 519 | 44.00 | 2.274 | 1187 | 6.730 | 14.86 |
| 556 | 32.56 | 3.068 | 1217 | 6.532 | 15.31 |
| 562 | 31.70 | 3.155 | 1242 | 6.388 | 15.64 |
| 612 | 23.98 | 4.170 | 1261 | 6.304 | 15.86 |
| 674 | 18.88 | 5.390 | 1295 | 6.110 | 16.37 |
| 687 | 17.57 | 5.690 | 1329 | 5.968 | 16.75 |
| 723 | 15.43 | 6.480 | 1357 | 5.854 | 17.07 |
| 747 | 14.51 | 6.890 | 1378 | 5.752 | 17.38 |
| 755 | 13.94 | 7.175 | 1407 | 5.645 | 17.70 |
| 777 | 13.52 | 7.390 | 1431 | 5.540 | 18.04 |

370° C., the curve for the reciprocal of susceptibility shows a marked concavity upward till about 500° C. when there is an interval of 400° for which it is remarkably straight. Between 900° C. and 1000° C. it curves downward after which it continues straight up through the melting point. On this basis, there appear to be three distinct regions of beta nickel. It is interesting to observe that on passing through the melting point (1452° C.) there is no change whatever in the magnetic behavior.

[1] Burgess, Bulletin, Bureau of Standards, Vol. 10, p. 315, 1914.

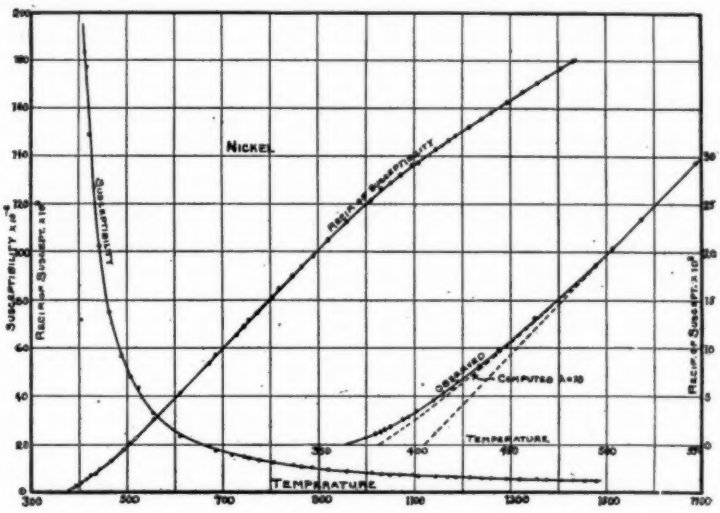



Fig. 5.

Susceptibility as a function of temperature for nickel.

The results for cobalt are given in Table IV. and plotted in Fig. 6. Beta cobalt is similar in its magnetic behavior to beta iron. Starting

TABLE IV.

| Temp. | $\chi \cdot 10^{-6}$. | $\frac{1}{\chi} \cdot 10^{4}$. | Temp. | $\chi \cdot 10^{-6}$. | $\frac{1}{\chi} \cdot 10^{4}$. |
|---|---|---|---|---|---|
| 1100 | 6010 | .01663 | 1450 | 54.48 | 1.836 |
| 1144 | 777 | .1287 | 1479 | 48.02 | 2.167 |
| 1178 | 403 | .2482 | 1490 | 35.42 | 2.824 |
| 1206 | 294.5 | .3395 | 1517 | 32.10 | 3.130 |
| 1237 | 224.7 | .4450 | 1528 | 29.10 | 3.436 |
| 1267 | 161.2 | .6205 | 1533 | 25.95 | 3.854 |
| 1296 | 133.2 | .7508 | | | |
| 1327 | 104.7 | .9615 | | | |
| 1331 | 102.0 | .9800 | | | |
| 1362 | 88.4 | 1.131 | | | |
| 1396 | 72.08 | 1.387 | | | |
| 1416 | 63.92 | 1.564 | | | |

with the relatively high transformation point of 1100° C. the curve for the reciprocal of susceptibility has a gradual upward curvature throughout the entire region up to the melting point (1485° C.). At this point, the susceptibility curve shows a slight knick, continuing regular, however, as far as the observations were carried.

*Discussion of Results.*—These results are in marked contrast to those

obtained by Weiss and Foëx,[1] who found Curie's law to hold in all cases. For beta iron, cobalt, and nickel at low temperatures they found, however, that the points for the reciprocal of susceptibility against temperature do not lie on a single straight line but on two straight lines with well defined points of intersection; *e. g.*, 828° C. for iron, 412° C. for nickel, and 1241° C. for cobalt, and they accordingly speak of $\text{beta}_1$ and $\text{beta}_2$ regions. My experiments have been repeated a great many times and in no case have I ever found evidence of two straight lines but always a smooth curve. For gamma iron, their departures from Curie's law were greatest but there seemed to be a tendency to obey it while my results show that not only is Curie's law not obeyed but the small curvature in the susceptibility plot is in the opposite direction. Delta iron was beyond the temperature range of their apparatus and they were unable to determine the shape of the curve in this region. The transformation temperature which I have obtained, 1407° C., is eight degrees higher than the value given by them. It is obviously then impossible to deduce a Curie constant for these metals or assign a definite number of magnetons to the atom according to the Weiss theory.

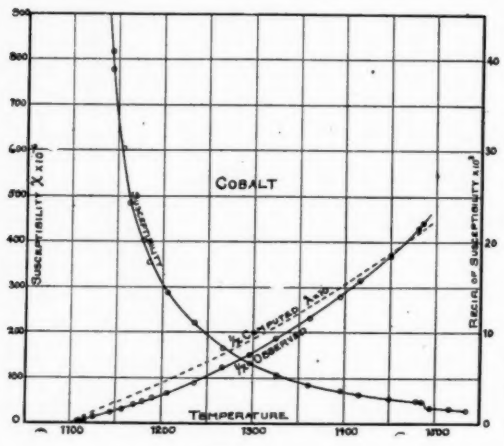



Fig. 6.

Susceptibility of cobalt as a function of temperature.

*Keesom's Quantum Theory of Magnetism.*—Keesom,[2] following the method of Debye[3] for specific heats, has introduced the quantum theory into the theory of magnetism and obtained an expression for the variation of susceptibility with temperature which, while it does not represent the facts accurately, gives a much better agreement than any other theory thus far proposed. He supposes that the amount of thermal energy which the molecules possess at a given temperature $T$ is to be obtained from a consideration of the equilibrium of a system of rotating molecules with black radiation at that temperature and assumes that Planck's formula, with the addition of a zero point energy, gives the rotational energy in the same manner, as for a linear oscillator. He supposes

[1] Weiss and Foëx, Arc. des Sc. Phys. et Nat., Vol. XXXI., p. 89, 1911.

[2] Communications, Phys. Lab. Leyden. Sup. No. 32.

[3] Debye, Ann. der Phys., Vol. 39 (4), p. 789, 1912.

further that the molecular rotational energy in a substance can be resolved into a system of stationary waves with a finite number of frequencies and that the high frequency limit is obtained by putting the total number of modes of vibration equal to the number of degrees of freedom, and that the velocity of propagation for each mode of vibration is proportional to the square root of its energy. Taking Planck's equation for the energy of rotation of a single molecule of frequency $\nu$, where by frequency is understood the number of rotations per second,

$$U_{r\nu} = \tfrac{1}{2}\left\{ \frac{h\nu}{e^{h\nu/KT} - 1} + \tfrac{1}{2}h\nu \right\} \tag{2}$$

we have for the mean rotational energy for two degrees of freedom about two equivalent axes

$$U_r = \frac{3}{\nu_{\max}{}^3} \int_0^{\nu_{\max}} \left\{ \frac{h\nu}{e^{h\nu/KT} - 1} + \tfrac{1}{2}h\nu \right\} \nu^2 d\nu. \tag{3}$$

Introducting the auxiliary variables

$$\xi = \frac{h\nu}{KT}, \quad x = \frac{h\nu_{\max}}{KT}, \quad \theta = \frac{h\nu_{\max}}{K}, \quad \theta_0 = \frac{h\nu_{\max\,0}}{K}, \tag{4}$$

the following equations are obtained which determine $U_r$ as a function of $T$.

$$\left(\frac{U_r}{U_{r_0}}\right)^{\frac{1}{4}} = 1 + \frac{8}{x^4} \int_0^x \frac{\xi d\xi}{e^{\xi} - 1}, \tag{5}$$

$$\frac{T}{\theta_0} = \frac{1}{x}\left(\frac{U_r}{U_{r_0}}\right)^{\frac{1}{4}}, \tag{6}$$

where

$$U_{r_0} = \frac{3}{8} K\theta_0. \tag{7}$$

The right-hand member of (5) may be developed in the following power series which hold respectively for the larger and smaller values of $x$:

$$\left(\frac{U_r}{U_{r_0}}\right)^{\frac{1}{4}} = 1 + \frac{8\pi^4}{15}\frac{1}{x^4} - 8\sum_{n=1}^{n=\infty} e^{-nx}\left\{ \frac{1}{nx} + \frac{3}{n^2x^2} + \frac{6}{n^3x^3} + \frac{6}{n^4x^4} \right\}, \tag{8}$$

$$\left(\frac{U_r}{U_{r_0}}\right)^{\frac{1}{4}} = \frac{8}{3}\frac{1}{x}\left\{ 1 + \frac{1}{20}x^2 - \frac{1}{1680}x^4 + \frac{1}{90720}x^6 - \cdots \right\}. \tag{9}$$

In the introduction of the quantum theory into the Weiss theory, it is supposed that the only change brought about is that in discussing the statistics of the orientations of the elementary magnets, the expression $RT$ for the energy of thermal agitation used in the relation deduced by Langevin is to be replaced by the value $U_r$ for the rotational energy.

Thus the Langevin expression for the intensity of magnetization of a paramagnetic gas

$$\frac{I_v}{I_{v_0}} = \frac{\cosh a}{\sinh a} - \frac{1}{a} \quad \text{where}^1 \quad a = \frac{\mu H}{RT} \tag{10}$$

is modified only by putting $a$ equal to $\mu H/U_r$. In the above expression, $I_{v_0}$ is the intensity of magnetization supposing no thermal agitation present and $\mu$ is the moment of a single rotating magnetic molecule.

The particular region in which we are interested in this work, namely, that immediately above the Curie point, has been called by Weiss the region of induced ferromagnetism and is characterized by the fact that the molecular field is brought into existence only by the action of an external field and is not so large that the external field may be neglected in comparison to it as is the case in the ferromagnetic region. Accordingly, we have, for the total field of equation (10)

$$H = H_e + NI_v, \tag{11}$$

where $N$ is the molecular field coefficient which is considered constant for a given substance.

The spontaneous magnetization $I_v$ which corresponds to an external field $H_e = 0$, is obtained by the well-known Weiss construction by obtaining the intersection of the curve of equation (10) and the straight line

$$\frac{I_v}{I_{v_0}} = \frac{U_r}{\mu N I_{v_0}} a, \tag{12}$$

which corresponds to the particular value of $T$ chosen. The magnitude of the rotational energy $U_{rc}$ at the Curie point is obtained by equating the slope of the curve at the origin with that of the straight line. Accordingly,

$$\frac{U_{rc}}{\mu N I_{v_0}} = \frac{1}{3}, \tag{13}$$

whence

$$U_{rc} = \frac{\mu N I_{v_0}}{3} = \frac{n\rho\mu^2 N}{3}, \tag{14}$$

[1] Before proceeding further it is necessary to define clearly certain of the terms entering into the discussion which are frequently confused by writers in this field.

$I_v$ equals vol. intensity of magnetization, = magnetic moment/volume,

$I_m$ equals mass intensity of magnetization, = magnetic moment/mass,

$K$ equals vol. susceptibility, $= I_v/H_e$.

$\chi$ equals mass susceptibility, $= I_m/H_e$;

accordingly

$$I_v = \frac{M}{v} = \frac{\rho M}{m} = \rho I_m$$

and $k = \rho\chi$.

Where $\rho$ equals density. In equation (11) it is the volume intensity which is meant.

where $n$ is the number of rotating molecular magnets per gram and $\rho$ the density of the substance.

The relation between the energy of rotation and the susceptibility is obtained as follows:

$$\chi = \frac{I_m}{H_e} = \frac{I_m}{H - H_m} = \frac{I_v}{\rho(H - NI_v)}, \tag{15}$$

$$\frac{1}{\chi} = \rho\left\{\frac{H}{I_v} - N\right\} \tag{16}$$

but from (10) for this particular region

$$\frac{H}{I_v} = \frac{3U_r}{I_{v_0}\mu} = \frac{3U_r}{n\rho\mu^2}. \tag{17}$$

Therefore

$$\frac{1}{\chi} = \frac{3}{n\mu^2}\{U_r - U_{rc}\}. \tag{18}$$

In the first approximation, namely, for equipartition, and also in the second approximation when, in the development according to positive powers of $x$, of the second member of equation (5) only the first powers are used, this expression goes over into Weiss's law for the susceptibility above the Curie point. When, however, the complete development is used, the relation between $\chi$ and $T$ is as shown in Fig. 7.

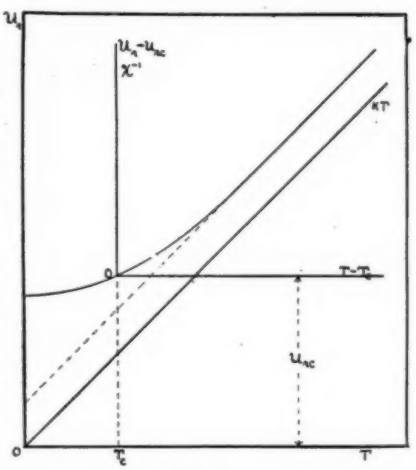



Fig. 7.

*Application of Theory to Experimental Results.*— Before the theory can be put to test it is necessary to determine the equation for the curve of Fig. 7. The problem algebraically consists in obtaining $U_r$ as a function of $T$ from equations (5) and (6) and substituting in equation (18). While it is impossible to write out this equation in analytical form, one may compute the theoretical curve by means of the power series (8) and (9). For simplicity write equation (18) in the form

$$\chi^{-1} = AU_r - B, \tag{19}$$

where $A = 3/n\mu^2$, and $B = N\rho$.

Put

$$\left(\frac{U_r}{U_{r_0}}\right)^4 = 1 + \frac{8}{x^4}\int_0^x \frac{\xi^3 d\xi}{e^\xi - 1} = f(x). \tag{20}$$

Then

$$\chi^{-1} = \alpha f^2()x + \beta, \tag{21}$$

where $\alpha = A U_{r_0}$, $\beta = -B$.

$$T = \gamma \frac{f(x)}{x}, \tag{22}$$

Whence

$$\frac{\chi^{-1} - \beta}{\alpha} = f^2(x) = n(y), \tag{23}$$

$$\frac{T}{\gamma} = \frac{f(x)}{x} = y. \tag{24}$$

Thus the experimental $\chi^{-1} - T$ curve is directly related to the experimental $\phi(y) - y$ curve by means of the three parameters $\alpha$, $\beta$, and $\gamma$. From the developments of equations (8) and (9), the following table was computed.

TABLE V.

*Calculation of Theoretical Curve.*

| $x$. | $f(x)$. | $f^2(x)$. | $\frac{f(x)}{x}$. |
|---|---|---|---|
| .250 | 10.702 | 114.5 | 42.81 |
| .500 | 5.486 | 30.09 | 10.97 |
| 1.000 | 2.799 | 7.838 | 2.799 |
| 1.250 | 2.297 | 5.275 | 1.838 |
| 1.429 | 2.052 | 4.213 | 1.436 |
| 1.667 | 1.815 | 3.292 | 1.088 |
| 2.000 | 1.588 | 2.523 | .794 |
| 2.500 | 1.378 | 1.897 | .5508 |
| 3.000 | 1.252 | 1.566 | .4172 |
| 3.200 | 1.216 | 1.478 | .3800 |
| 3.333 | 1.1956 | 1.428 | .3588 |
| 3.400 | 1.1856 | 1.406 | .3488 |
| 3.600 | 1.1604 | 1.346 | .3225 |
| 3.800 | 1.1388 | 1.297 | .2998 |
| 4.000 | 1.1212 | 1.257 | .2804 |
| 4.200 | 1.1056 | 1.221 | .2632 |
| 5.000 | 1.0628 | 1.131 | .2127 |
| 6.667 | 1.0238 | 1.049 | .1535 |
| 10.000 | 1.0051 | 1.010 | .1005 |

The $y$, $\phi(y)$ curve was then plotted, which is, in reality, the theoretical curve of Fig. 7. The direct method of obtaining the constants $\alpha$, $\beta$, and $\gamma$, is by means of the slope of the asymptote and its intercept on the $\phi(y)$ axis, and this method was at first employed. A simpler method, sufficiently accurate for the purpose, is to determine $\gamma$ by means of the point on the curve at which departure from the asymptote begins which then fixes the correspondence of $T$ and $y$. Then, by getting corresponding

values of $\chi^{-1}$ and $\phi(y)$ for two different values of $T$, two equations in $\alpha$ and $\beta$ are obtained and their values computed therefrom.

When the calculations were carried out in this manner and applied to the results for nickel, it was found that, although the theoretical curve possesses the same general shape as the experimental one, the departure from the asymptote is not sufficiently rapid. An attempt was then made to see whether by the assumption of a larger zero point energy than is taken in Planck's equation, a better agreement with experimental results could be obtained. This was done by introducing in equation (2) the variable parameter $\lambda$ as follows:

$$U_{r\nu} = \tfrac{1}{2}\left\{ \frac{h\nu}{e^{h\nu/KT} - 1} + \frac{\lambda}{2} h\nu \right\}. \tag{25}$$

When this modified expression is used, and the analytical work carried through in a manner analogous to that employed by Keesom, the following modified developments result:

$$\left(\frac{U_r}{U_{r_0}}\right)^4 = 1 + \frac{8\pi^4}{15\lambda x^4} - \frac{8}{\lambda}\sum_{n=1}^{n=\infty} e^{-nx}\left\{ \frac{1}{nx} + \frac{3}{n^2x^2} + \frac{6}{n^3x^3} + \frac{6}{n^4x^4} \right\}, \tag{26}$$

$$\left(\frac{U_r}{U_{r_0}}\right)^4 = 1 - \frac{1}{\lambda} + \frac{1}{\lambda}\frac{8}{3x}\left\{ 1 + \frac{1}{20}x^2 - \frac{1}{1680}x^4 + \frac{1}{90720}x^6 \cdots \right\}, \tag{27}$$

which were then used in connection with equations (5) and (6).

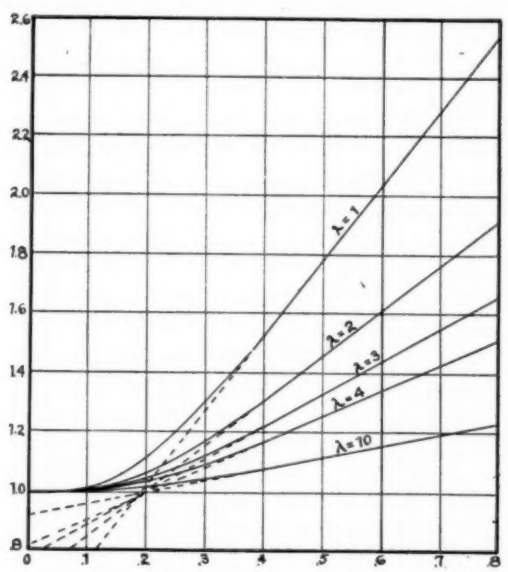



Fig. 8.

By assigning different values to $\lambda$ the family of $y$, $\phi(y)$ curves shown in Fig. 8 was computed. These curves were then transformed so as to

have the same asymptote with their points of departure from the asymptote made to coincide, and the family shown in Fig. 9 was computed. Taking $\lambda$ equal to ten and fitting once more to the experimental curve for nickel, the curve shown dotted in Fig. 5 was obtained. It is to be observed that the agreement is still unsatisfactory. A better agreement

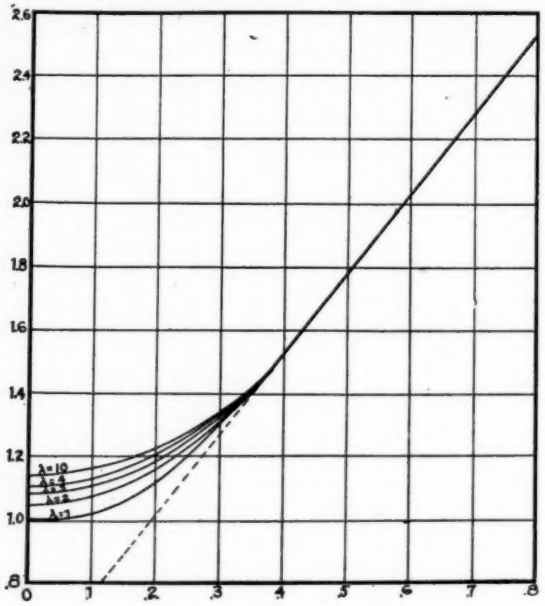



Fig. 9.

would of course result if a still larger value of $\lambda$ were used; but with the value employed, the theory is so badly distorted that it does not seem worth while. The experimental curve for cobalt shows no tendency to approach an asymptote below the melting point hence it is difficult to attempt to check it. However, selecting the region of the theoretical curve for which the curvature is greatest, to give the theory the best possible advantage, the dotted curve in Fig. 6 was obtained. The departures between the theoretical and experimental curves for beta iron are so great that no calculation was made.

## SUMMARY

1. The magnetic behavior of iron, nickel, and cobalt has been studied from the temperatures of their transformation points up through their melting points.

2. With the exception of nickel in limited regions, Curie's law is not obeyed.

3. In the regions immediately above the transformation points, for all three metals, the curve for the reciprocal of susceptibility is a smooth one with a marked upward concavity, instead of two intersecting straight lines as obtained by Weiss and Foëx.

4. Keesom's quantum theory of magnetism fits the experimental results in a qualitative way, but requires further modification.

5. The assumption of a larger zero point energy in Planck's equation is not sufficient to bring the theory into agreement with experimental facts.

PHYSICAL LABORATORY,
UNIVERSITY OF WISCONSIN,
December, 1916.

# THE MEASUREMENT OF MAGNETIC PERMEABILITY.

BY ARTHUR WHITMORE SMITH.

IT is well known that when a ring of iron is magnetized by a current in a surrounding solenoid the iron does not instantly reach its full magnetization, but after the first large increase the magnetization continues to increase, in some cases, for several seconds. Ewing[1] observed this "viscous lagging" which "occurs in the softest iron and near the beginning of the steep part of the magnetization curve." Again he states[2] that this effect is more marked in soft iron than in hard iron, that it depends on the size of the sample tested, and that the cause is obscure. He also gives curves[3] showing the effect of increasing the magnetizing force by small steps. At each step there was an immediate increase of magnetization, followed by a slow increase to the final value.

Lord Rayleigh[4] tried to balance the effect on a magnetometer needle due to the magnetization of a bar of iron against the effect due to the current in a coil. A steady balance was impossible. "Most of the anomalous action was over in 3 or 4 seconds—the final magnetic state was not attained until after 15 or 20 seconds."

Wilson[5] has noticed a large creeping up in the magnetic flux, even after one second. He states that the "effect is peculiar to the iron itself" and "might be influenced by induced currents." He computes (erroneously, I think) that the eddy current effect had entirely subsided in 0.9 second. More recently Jouaust[6] has shown that the step-by-step method may give results as much as 12 per cent. in error, and he gives the warning that the sample tested should not be too thick. Rucker[7] has found considerable difference between measurements by the ring ballistic method and magnetometer measurements. Using electrolytic iron Taylor[8] has shown that owing to "magnetic viscosity" the step-by-

[1] Phil. Trans., Vol. 176, p. 569, 1885.
[2] Ewing, Magnetic Induction in Iron.
[3] Proc. Roy. Soc., Vol. 46, p. 269, 1889.
[4] Phil. Mag., Vol. 23, p. 230, 1887.
[5] Proc. Roy. Soc., Vol. 62, p. 375, 1897–8.
[6] Comp. Rend., Vol. 139, p. 272, 1904.
[7] Zeit. für Instrumentenkunde, Vol. 25, p. 354, 1905.
[8] PHYS. REV., Vol. 23, p. 95, 1906.

step method gives values for the magnetic induction considerably lower than those obtained by the method of reversals.

The question naturally arises, how much of the changing flux is measured by the ballistic galvanometer in the usual magnetic measurements? Such a galvanometer, having a period of about 16 seconds, will have completed its first throw in 4 seconds, and certainly cannot measure any change occurring after that time. At the very least, doubt is cast upon measurements made with this instrument under such conditions. The present paper shows that even the method of reversals with an ordinary ballistic galvanometer gives results which are too low, and the amount of this discrepancy for a ring of Swedish iron is shown in Fig. 2.

Some time ago there came into my hands a ballistic galvanometer having one remarkable characteristic. It is of the D'Arsonval type, period 40 seconds, and about as sensitive as other high-grade ballistic galvanometers, the remarkable property being that it is critically damped on a circuit of 70,000 ohms. When used on a circuit of a few ohms it is so greatly overdamped that it can move but slowly, and when once deflected it requires about 13 minutes to drift half way back to its zero position. It will therefore stand practically at rest at any point where it may be left, although it has a perfectly definite zero to which it will return in time. The action of this galvanometer is particularly interesting when it is connected to a few turns of wire around a bar magnet. When the magnet is slowly withdrawn, the spot of light on the galvanometer scale also moves slowly. If the magnet stops, so does the spot of light. When the motion is resumed the spot of light follows, and if the magnet be suddenly returned to its original position the spot of light returns as quickly and stops as suddenly. Every movement of the magnet is reproduced as faithfully as though the mirror were carried on the magnet itself. Of course, if the galvanometer is kept deflected too long there is the slow drifting back towards zero that was mentioned above, but if the movement is stopped with the galvanometer near its natural resting point it will remain stationary indefinitely. Perhaps the action of this galvanometer is best explained by saying that for every maxwell of flux cut by the wire around the magnet the coil of the galvanometer must move enough to cut one maxwell in the opposite sense. The instrument is, then, a fluxmeter rather than a galvanometer.

With the ordinary ballistic galvanometer the sudden reversal of the flux to be measured gives an impulse to the moving coil, and the scale is read at the end of the first throw. In using the new galvanometer the magnetic flux to be measured is reversed as usual, giving a quick deflection, at the end of which is a slow increase combined with the

return drift towards zero. After it has drifted back for half a minute or longer and has reached some definite line on the scale, the magnetic flux is reversed again, deflecting the galvanometer back to zero or a few spaces beyond. The slow increase is noticeable but the drift is negligible because the galvanometer is so near its resting point. After allowing about half a minute for the flux to reach its final value the scale is read, and this return throw is called the "deflection."

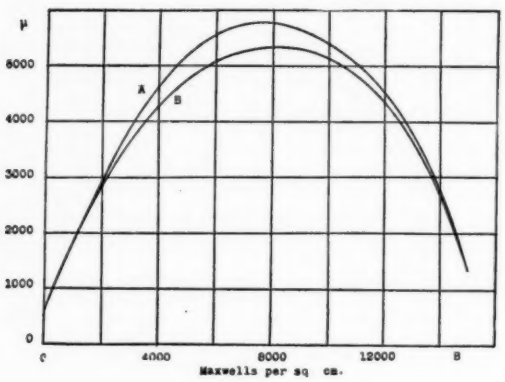



Fig. 1.

Permeability of Swedish iron. *A*. By new galvanometer. *B*. By ballistic galvanometer.

In order to compare this new galvanometer with a D'Arsonval ballistic galvanometer of the usual type, both instruments were used to measure the same flux. A ring of Swedish iron was magnetized beyond saturation and the magnetizing current then reduced by small steps, it being slowly reversed 10 or 12 times at each step. At each point the flux through the ring was measured by each galvanometer, using first one and then the other. Both galvanometer scales were carefully calibrated over the range in which they were used, the readings for the calibration being interspersed with the other readings to avoid any error due to a change in the sensitiveness of the galvanometers. At the highest magnetization each galvanometer showed the same values for the magnetic flux. At lower magnetizations the values of the flux as measured by the new galvanometer were considerably larger than the values given by the ordinary galvanometer. The values of the permeability computed from these two sets of measurements are shown by the curves *A* and *B*, Fig. 1. The higher curve is doubtless correct, and the lower curve shows that much is missed by the old type of gal-

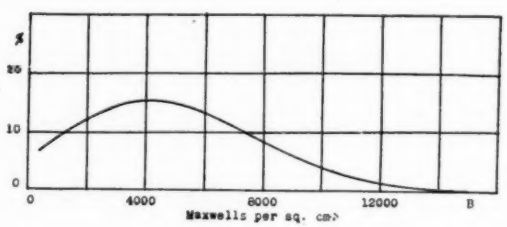



Fig. 2.

Percentage of magnetic flux not measured by the ballistic galvanometer (for a given value of H). For a ring of Swedish iron, 2 sq. cm. in cross section.

vanometer. The same thing is shown in a different way in Fig. 2, where the amount of flux not measured by the ballistic galvanometer is plotted as a percentage of the whole flux, and it is seen to be over 15 per cent. for values of $B$ about 4,000 maxwells per square centimeter.

The measurements were repeated at different times and the same values were obtained at each trial. The amount missed by the ballistic galvanometer is approximately equal to that measured by the galvanometer when its key is closed later than the reversal of the current by a period equal to the time required for the first swing of the galvanometer.

There are two principal reasons why the magnetization lags thus behind the reversal of the magnetizing current. In the first place the primary current does not reach its full value instantly owing to the large self-inductance of the circuit with its many turns around the iron ring. When there is little other resistance in the circuit, and a battery of low E.M.F. is used, this slow rise of the current can be easily observed with an ordinary ammeter. "The time needed to establish a current of given strength in the coil of a large electromagnet with a solid core may be several minutes."[1]

But in the results shown by the curves above the current was furnished by a storage battery of 35 volts and rose to its full value in a small fraction of one second. However, it was very evident that a considerable part of the magnetization was late, and occurred after the current had reached its steady value.

In the case of solid pieces of iron it is certain that there will be some current induced in the mass of the iron itself. This current will be greatest when the magnetization is increasing the fastest, and it will circulate in a direction opposite to that of the primary current. The effect of this eddy current (while it lasts) is, then, to reduce the resultant magnetic field which is impressed upon the iron, and of course the magnetic flux cannot increase faster than the impressed field.

To obtain more information regarding the cause of this slow magnetization a thin ring was cut from the side of the ring used above. All of the measurements were repeated with this smaller ring, which had one eighth the cross section of the former ring, but now both galvanometers gave the same results. The permeability curve is shown at $A$, Fig. 3. Only one curve is shown as the width of the line is sufficient to cover both curves obtained by the two galvanometers. This result was expected, but it was not expected that the permeability of the small ring should drop to half that of the piece from which it was cut. It was therefore unwound and heated to a dull red and allowed to cool slowly.

[1] B. O. Peirce, Am. Acad. Arts and Sci., Vol. 43, 1907, p. 116.

After this annealing it was again tested in the same manner as before and the results are shown by curve *B*. As before the width of the line covers the two curves obtained by the two galvanometers. The permeability has nearly been restored and doubtless would be larger if greater care had been taken in the annealing. There was some creeping of the magnetization, but the galvanometer key had to be closed one fifth of a second after the reversal of the current to detect as much creeping as was found after four seconds with the larger ring. It therefore seems reasonable to suppose that the slow increase of the magnetization is due to eddy currents in the iron of the ring, these currents tending to shield the interior portions of the iron from the changes in the field impressed by the current in the surrounding solenoid. This agrees with the usual comment that the amount of the creeping depends upon the size of the sample studied and is absent in fine wires.

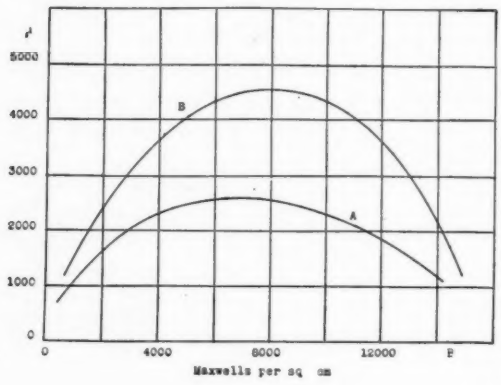



Fig. 3.

Permeability of Swedish iron. Points by both galvanometers are covered by the width of the lines. *A*. Thin ring cut from larger one. *B*. Same ring after annealing.

It is also interesting to see what very marked changes are produced in the permeability by the mere cutting of the iron in the lathe, an effect which must be present more or less in every sample that is thus prepared.

It follows from this comparison of galvanometers that the ordinary ballistic galvanometer does not give the correct measurement of magnetic flux even when the sample tested is only two or three square centimeters in cross section, especially if the permeability of the iron is high. In this case it is necessary to use an instrument which will respond quickly to changes of magnetic flux in order that the observations may not be too tedious, but which at the same time will have a period considerably longer than the time required for the iron to reach complete magnetization.

PHYSICAL LABORATORY,
UNIVERSITY OF MICHIGAN,
January, 1916.

# TIME LAG IN MAGNETIZATION.

BY ARTHUR WHITMORE SMITH.

WHEN a steady magnetic field is suddenly impressed on a bar of iron the outside of the iron becomes magnetized first, while the center of the bar follows later in something the same way that the temperature would rise when the bar is plunged into a hot bath. Let us consider a long bar of iron, magnetized by a steady current flowing through a uniform winding around the bar. Let $B'$, $H'$, $\mu'$ denote the values of $B$, $H$, and $\mu$ at the center of the bar, where the time lag would certainly be the greatest.

The magnetizing force due to the current is $H_0 = 4\pi I$, where $I$ denotes the C.G.S. current turns per cm. If there is also an eddy current of $J$ C.G.S. units per cm. length of the bar the resultant field at the center is

$$H' = 4\pi(I - J). \tag{1}$$

To find $J$ let us consider a cylindrical portion of the bar, of radius $r$ and of thickness $dr$. The current circulating in unit length of this tube is

$$dJ = \frac{e dr}{2\pi r \sigma}, \tag{2}$$

where $\sigma$ denotes the specific resistance and $e$ the E.M.F. induced in this tube. This E.M.F. is due to the changing magnetic flux within the tube.

The simplest case is that in which the value of $B$ is the same at all points within the bar. This would give for the flux within any given tube,

$$\Phi = \pi r^2 B' \tag{3}$$

and the E.M.F. induced in the tube is, then,

$$e = \frac{d\Phi}{dt} = \pi r^2 \frac{dB'}{dt}. \tag{4}$$

This gives

$$J = \frac{1}{2\sigma} \frac{dB'}{dt} \int_0^a r dr = \frac{a^2}{4\sigma} \frac{dB'}{dt}, \tag{5}$$

since $dB'/dt$ is independent of $r$.

Substituting this value of $J$ in (1)

$$H' = H_0 - \frac{\pi a^2}{\sigma}\frac{dB'}{dt} \tag{6}$$

or, since

$$\frac{dB'}{dt} = \frac{dB'}{dH'}\frac{dH'}{dt} = \mu'\frac{dH'}{dt}, \tag{7}$$

$$H' = H_0 - \frac{\mu'\pi a^2}{\sigma}\frac{dH'}{dt}. \tag{8}$$

An approximate solution of this differential equation can be obtained by taking $\mu'$ as constant, in which case

$$H' = H_0(1 - \epsilon^{-(\sigma t/\mu'\pi a^2)}). \tag{9}$$

This derivation is of interest as showing why there is an eddy current, and upon what factors it depends. Particularly should it be noticed that $\mu'$ is a kind of differential permeability, its value being given by the slope of the curve which shows the relation between $B'$ and $H'$. The approximations used above result in too small a value for the numerical coefficient of the exponent, and owing to the eddy current thus produced $H$ will not be uniform across the bar.

Using the more general equations of Maxwell's theory, Lord Rayleigh[1] showed that the eddy current in a cylindrical conductor decreases with time as $\epsilon^{-(1.44\sigma t/\mu'\pi a^2)}$, which gives 1.44 as the coefficient of the exponent. This result is often quoted to show that the effect of eddy currents dies out too quickly to account for much of the observed "magnetic viscosity," although it is generally remarked that this effect is greater in iron of large cross section and is most noticeable in the region of maximum permeability.

J. J. Thomson[2] derived a similar expression and computed that for an iron rod of 1 cm. radius and $\mu = 1{,}000$ the induced current will fall to $1/\epsilon$ of its maximum value in 2/9 of a second. For soft iron $\mu'$ may be as large as 12,000, in which case 1 per cent. of the eddy current would still remain after 10 seconds, and of course there will be some eddy current as long as $B$ is changing.

Ewing explains[3] this time lag of magnetization as largely due to the smaller relative amount of surface on the larger bars and the longer chains of molecular magnets. An outlying molecule is first upset; then its neighbors, weakened by the loss of its support, follow suit and the action spreads from molecule to molecule. The surface molecules are least securely held. In fine wire there is much surface in proportion

[1] B. A. Report, 1882, p. 446.

[2] Recent Researches, pp. 352–6.

[3] Magnetic Induction, p. 335.

to the amount of iron, and the breaking up of molecular communities starts at so many points that it is completed very quickly.

In order to examine further the effect of eddy currents, four rings of Swedish iron were made. The cross sectional area of each was about 1 cm. by 1.2 cm. Two of the rings had an external diameter of 15 cm., while the internal diameter of the other two was a little larger. When placed together the four rings formed a ring of larger cross section (2.1 cm. by 2.5 cm.), but with the four parts separated by air gaps of about 1 mm. in width. These air gaps could have no effect upon the magnetization of the ring, but they do prevent any eddy current circulating through the ring as a whole; there can only be smaller eddies in the four parts. The magnetic characteristics of this composite ring are shown by the hysteresis curve *A*, Fig. 1. The hysteresis curves for each of the four separate rings are likewise represented, by this same curve, which was obtained by the method of direct deflection[1] but using the fluxmeter-galvanometer described in the previous paper.

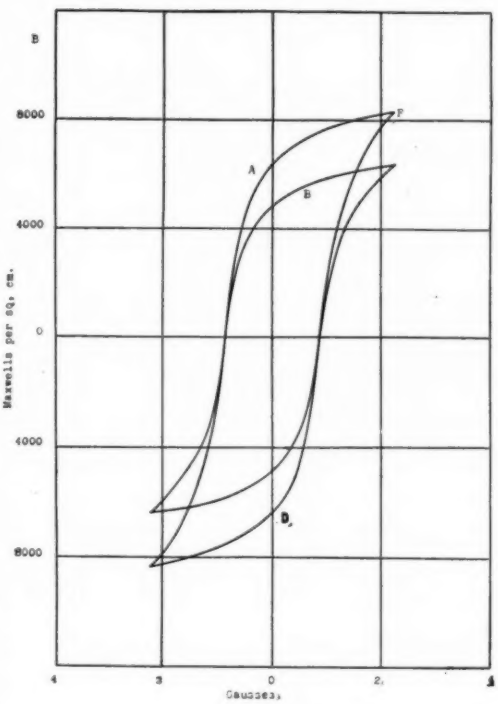



Fig. 1.

Hysteresis of Swedish iron. *A*. Four separate rings. *B*. After soldering together.

The time lag was studied by means of a pendulum apparatus which closed the galvanometer key at any desired interval after closing the battery circuit. A storage battery of 125 volts was used to reduce the time lag of primary current to a negligible amount. The magnetizing current of 0.25 ampere ($H = 2.2$ gausses) was reversed slowly (once every ten seconds) many times, and the circuit left open. The pendulum then swung, closing the battery circuit so as to continue the magnetic cycle from *D* to *F*, Fig. 1, and then, after a predetermined interval, closing the galvanometer key. The galvanometer deflection measures the amount of change in the magnetic flux after the key is closed, that

[1] A. W. Smith, Electrical Measurements, p. 182.

is, the amount by which the flux lacked of reaching its final value at the moment the key was closed. As this galvanometer responded promptly to every change of flux, but moved almost not at all under its own restoring forces, the total change of flux was accurately measured even when spread over considerable time. The growth of the flux is shown by the curve *P*, Fig. 2, the width of the line covering all of the

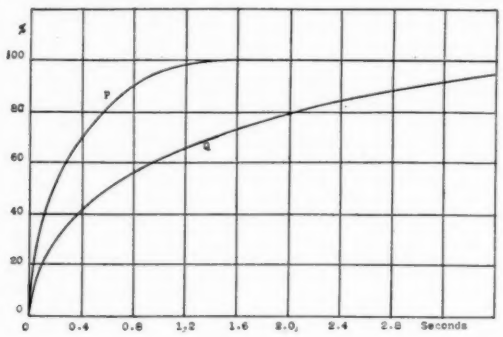



Fig. 2.

Time lag of magnetization. (H = 2.2 gausses.) *P*. Four separate rings. *Q*. Four rings soldered together.

fourteen observed points. The ordinates are expressed in per cent. of the final value, and the abscissæ are the times in seconds after closing the primary circuit. The curve shows that the magnetic flux has reached its final value after about 1.4 seconds.

Now if this lag in magnetization is due to the magnetic viscosity of the iron it should make no difference if the air gaps separating the four rings should become conducting electrically. Neither would this change the amount of iron surface bounding the four rings if the conducting gaps are non-magnetic.

In carrying out this idea copper rings were fitted tightly between the iron rings and the whole soldered together into one solid ring. This ring was rewound with the same number of turns as before, and all of the measurements were repeated. Evidently the ring had received some treatment which lowered its permeability and this must be taken into account if the measurements made on it are to be comparable with those shown by curve *P*. The hysteresis curve is shown at *B*, Fig. 1, and shows the decreased permeability. In spite of this, the time lag of magnetization was very much more marked than before. The results are shown by curve *Q*, Fig. 2, in which the ordinates are per cent. of the final value as before, and the abscissæ are the observed times corrected to give the same values of $t/\mu'$ (see (9)) as for curve *P*. It is assumed that the specific resistance of the iron is the same as before. For a straight bar of the same kind of iron this was found to be 11,000 C.G.S. units.

Comparing these two curves it is seen that in $P$ the magnetization rises to 0.8 of its final value in 0.58 second, while in $Q$ it rises much slower, taking 2.05 seconds to reach 0.8 of the final value. This is nearly a fourfold increase, which was expected since the cross section is increased four times. It is evident that these two curves are not strictly of the form $\epsilon^{-kt}$, and this is not to be expected since $\mu$ has a considerable range while the iron is going over the quarter cycle from $D$ to $F$. The maximum value of $B$ for this cycle is 8,300, corresponding to $H = 2.2$; this would give 3,760 for the ordinary value of the permeability. But the total change in $B$ corresponding to the change in $H$ from 0 to 2.2 is 14,700, giving $\mu' = 6{,}700$. As any given portion of iron is carried along the cycle from $D$ to $F$, the value of $\mu'$ at $D$ is about 2,000, increasing at the steepest part of the curve to 14,000 and then decreasing to about 2,200 at $F$. This is also shown in the curves as they rise at first more rapidly than exponential curves, then more slowly, and at the end they quickly reach the final value. Moreover the iron at different points in the ring is at different stages of magnetization and at any given instant no value of $\mu$ could be appropriate for all portions of the iron. These curves show the increase in the total flux for the entire ring. Equation (9) with the value of $\mu' = 4{,}000$ agrees with the middle portions of these curves. The magnetic induction at the center of the iron rises much more slowly, probably not more than half as fast, and the average value of $\mu'$ could easily be as much as 8,000.

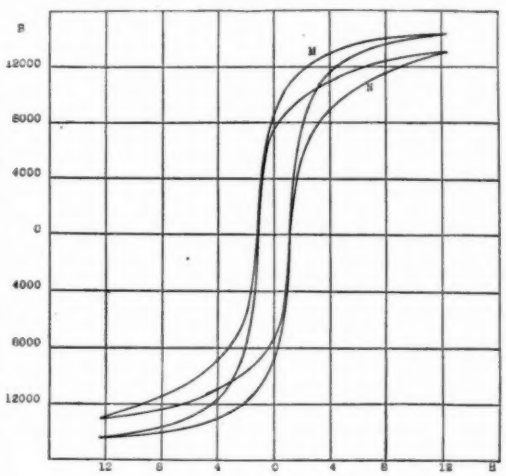



Fig. 3.

Hysteresis of Swedish iron. *M*. Four separate rings. *N*. After soldering together.

A ring of cast iron was examined in the same way and almost no time lag was found. The formula would also lead one to expect such a result, since $\mu$ is small for cast iron and whatever time effect there might be would be over in a very short time.

But it is not necessary to use a different kind of iron to obtain a small value of $\mu$. The same rings of Swedish iron were subjected to a magnetizing force of 12.4 gausses, and the hysteresis curves are shown in Fig. 3. While the steepest part of the curve is much like that in Fig. 1, a large

part of it is flatter and at the end $\mu' = 300$. Thus on the whole $\mu'$ is less and the time lag of $H'$ would be less than in the former case. Moreover, the last 10 per cent. change in $H'$ would not have a corresponding effect in changing either $B'$ or the total flux measured by the galvanometer. The observed time lag for the four separate rings is shown by curve $R$, Fig. 4, in which the abscissæ are the observed times in seconds, the same as in curve $P$ for the same rings at lower magnetization. This curve corresponds to an average value of $\mu = 700$, and if $H'$ rose half as fast as this an average value of $\mu' = 1{,}400$ is not difficult to explain on the basis of the actual hysteresis curve $M$.

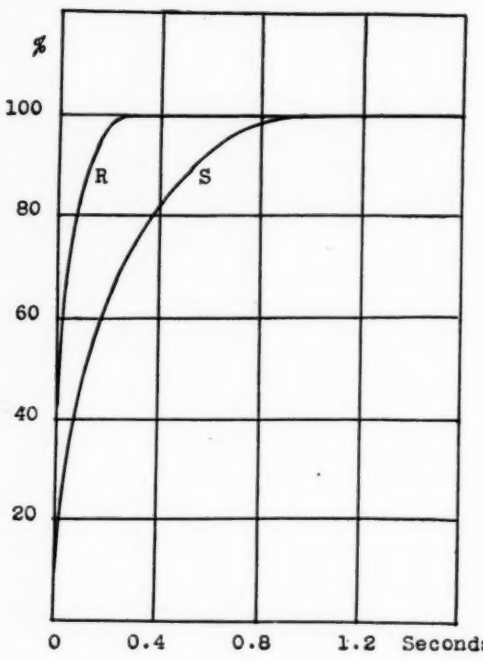



Fig. 4.

Time lag of magnetization. (H = 12.4 gausses.) *R*. Four separate rings. *S*. Four rings soldered together.

The hysteresis curve for these rings after they were soldered together, with copper filling the intervening spaces, is shown by $N$, Fig. 3. The lowered permeability is again in evidence but not as much as before. The time lag for this ring when $H = 12.4$ is shown by $S$, Fig. 4, where the curve has been made comparable with $R$ by increasing the observed abscissæ inversely as the permeabilities. The relation between $R$ and $S$ is about the same as between $P$ and $Q$, and the relation between $R$ and $P$, for the same rings, is what would be expected in view of the change in the average value of the permeability.

With more mathematical analysis it would be possible to show more precisely the relation between the flux through a bar of iron at any instant and the time which has elapsed since the steady field was applied. But the purpose of the present paper has been fulfilled by showing that the observed time lag in these rings can be accounted for by the eddy currents which the formula indicates should circulate in the iron. Further effects of these eddy currents will be given in another paper.

PHYSICAL LABORATORY,
UNIVERSITY OF MICHIGAN,
February, 1917.

# A STUDY OF THE ACCELERATED MOTION OF SMALL DROPS THROUGH A VISCOUS MEDIUM.[1]

BY W. A. SHEWHART.

NOT long ago Professor E. P. Lewis[2] developed a new method of determining the amplitude of a sound wave in air. It was necessary to assume that if we have a small particle falling freely through a viscous medium under the action of weight, and at the same time acted upon by a simple harmonic force at right angles to the vertical fall, the equations of the vertical and horizontal components of the motion may be written respectively,

$$mg = k\frac{dy}{dt}, \tag{1}$$

$$m\frac{d^2x}{dt^2} - k\frac{dx}{dt} = A \sin \omega t, \tag{2}$$

where $m$ represents the mass of the particle, $x$ and $y$ the displacement at any instant, $A$ the amplitude of the simple harmonic force, $g$ the acceleration of gravity and $k$ a constant based upon the assumption that the viscous resisting force, in both cases, bears the same proportion to the first power of the velocity. Three general questions are involved. In the first place what are the limitations under which equation (1) holds if the particle moves under the influence of a constant force in one dimension only? Stokes[3] has stated this problem as that of the motion of an incompressible fluid, infinite in extent and of uniform density, moving with a small velocity and without slipping, past the surface of a small sphere, and he arrives at the well-known formula,

$$F = 6\pi\mu a v, \tag{3}$$

where $F$ is the resultant viscous force, $\mu$ the coefficient of viscosity of the medium, $a$ the radius of the sphere and $v$ the terminal velocity. The fall of solid and fluid spheres in air and in liquids has been very carefully studied experimentally by Millikan,[4] Nordlund,[5] Jones,[6] Allen,[7]

[1] Accepted in partial satisfaction of the requirements for the degree of doctor of philosophy in the University of California.

[2] Lewis and Farris, PHYS. REV., N.S., Vol. VI., p. 492.

[3] Stokes, Math. and Phys. Papers, Vol. III., p. 57.

[4] Millikan, PHYS. REV., Vol. 4, Apr., 1911, p. 349; Vol. 2, Aug., 1913, p. 109.

[5] Nordlund, Zeitschrift für Physikalische Chemie, 87, 1914, p. 40.

[6] Jones, Phil. Mag., 37, p. 45, 1914.

[7] Allen, Phil. Mag., 1900, p. 323.

Zeleny,[1] Arhold,[2] Ladenburg,[3] Silvey,[4] Ellis,[5] and others, and the theoretical development given by Stokes has been extended and modified to fit certain conditions not contained in the earlier treatment. The work of these men shows that equation (1) represents the motion of a particle with a uniform linear velocity $V$, so long as the quantity $\frac{Va}{\nu}$ is small, where $a$ is the radius of the particle and $\nu$ is the kinematic coefficient of viscosity.[6]

In the second place, no record of experimental evidence has been found by the writer in justification of using equation (2) as an expression of the horizontal vibratory motion, even though the particle be confined to move in one dimension. Although in the early theoretical work of Stokes, he arrives at an equation for the force acting on a pendulum bob, assuming that it oscillates in a straight line through a viscous medium, and later, after investigating the problem of the small sphere moving in a similar medium, he points out that the expression for the resisting force on the spherical bob reduces to that for the force acting on a small sphere, which moves with uniform velocity, when the period of the pendulum approaches infinity. Thus far the two equations (1) and (2) have been discussed independently one of the other, as expressions representing only a one-dimensional motion, and it has been pointed out that we cannot justly assume equation (2) without further experimental evidence. There still remains the third question, *i. e.*, granted that equations (1) and (2) hold separately, when the motion is linear, will they still serve to express the component relations when the motion is two-dimensional? The experiment described below was designed to answer this third question, and to determine to what extent, if any, the terminal vertical velocity component of a small sphere will be changed if a simple harmonic force is applied at right angles to the original motion.

Charged particles are made to fall between two condenser plates connected to the terminals of a Thoradson transformer, and illuminated in such a way that a photographic trace of the path of each small sphere may be obtained. In this case equation (2) becomes

$$m\frac{d^2x}{dt^2} + k\frac{dx}{dt} = neX \sin \omega t, \tag{4}$$

where $m$ and $k$ have the same meaning as previously and $X$ is the potential gradient of the field, $e$ the elementary charge and $n$ the number of

[1] Zeleny, PHYS. REV., Vol. 30, May, 1910, p. 535.

[2] Arnold, Phil. Mag., 22, 6, p. 755.

[3] Ladenburg, Ann. der Physik, 22, p. 287 (1907), and 23, p. 447, 1907.

[4] Silvey, PHYS. REV., N. S., Jan. 1916, p. 106.

[5] Ellis, Phil. Mag., 6th Series, Vol. 29, p. 526.

[6] Rayleigh, Phil. Mag., Vol. 36, 1893, p. 365.

elementary charges on a given particle. The complete solution of this equation is

$$x = \frac{neX}{k\omega} \sin \delta \sin (\omega t - \delta) + Be^{-(kt/m)} + B_1, \tag{5}$$

where $\delta$ is the phase difference between the motion of the particle and the varying simple harmonic disturbance and $B$ and $B_1$ the integration constants.

The second term soon becomes negligible in the actual case, whereas the third term, which is a constant, merely determines the point about which oscillations take place, depending upon the phase of the driving force at the instant the field is turned on. After a short time, under these conditions, the particles would vibrate back and forth in a horizontal line, but for the fact that they are falling at the same time so that the trace becomes an approximate sine curve.

From equation (5) the maximum displacement from the mean position $x = B$ is seen to vary directly as the number of elementary charges on the falling drop and directly as the magnitude of the field gradient $X$, providing of course, that $k$ remains a constant for such motion. Thus it is possible to test in a very simple way the involved assumption in regard to $k$, since the displacement, $x$ and $y$, may be read from the plate and $ne$ and $X$ may be determined experimentally. From equation (5), neglecting the last two terms, the horizontal motion, after a very short interval of time, is practically expressible as

$$V = A \sin \omega t, \quad \text{where} \quad A = \frac{ne\,X \sin \omega t}{k\omega}. \tag{6}$$

The experimental arrangement was as follows: Two parallel brass plates, each 17.78 cm. × 3.81 cm. × .5 cm., were supported on an ebonite base. The distance between them was accurately fixed at 3.8 cm. by means of ebonite bars, and grooves in the base. The plates were then connected to the terminals of a transformer and the air condenser thus formed was set parallel to, and about 7 cm. from the lens of the camera. The particles were illuminated by a carbon arc enclosed in a sheet iron cylinder, and the angle between the direct rays of the arc and the perpendicular to the face of the camera was adjusted to approximately 28°. A large sheet of paper in which a narrow slit 2 mm. × 10 mm. had been cut, was arranged about an inch above the condenser so that when a water or oil spray was made with an atomizer above the paper, the small drops fell into the illuminated field near the center of the condenser.

Needle points were placed at the top of the condenser so that as the drops fell past them they would pick up one or more elementary charges

and then when the drops came between the plates they would oscillate back and forth as they fell, thus tracing a sine-like path. After careful focusing on a glass fiber, held for the instant at the center of the condenser but withdrawn after final adjustment, a spray was made above the paper and so soon as the first droplets entered the field the shutter was opened from ½ to 5 seconds. Since the image on the plate was very narrow, each plate could be withdrawn a small distance at a time and thus made to serve for several exposures.

A lens of 1.5 cm. aperture and .9 cm. focal length was used. It was found necessary to employ the very sensitive Seed Graflex plate and the largest stop was used for most of the work, but a very faint narrow trace could be obtained with a stop .5 mm. in diameter.

To exclude all light coming from other sources, it was found helpful to place a screen immediately in front of the condenser. In every case, the best photographic results were obtained when the particles fell so as to be viewed against a background in which there was no reflecting material of any kind. For instance, if the screen was placed too near the condenser, some light would be reflected from its surfaces into the camera and fog the plates, even though all screens and the condenser itself was painted black.

Owing to convection currents, very small particles soon drifted out of focus, so that in the present work only comparatively large drops were used. At first an X-ray tube was employed to furnish a constant supply of ions between the plates in the hope that the particle would pick up charges as they fell, which would, as we have seen from equation (6), result in a change $\Delta A$ of the amplitude $A$, which would bear an integral relation to the previous amplitude. The average potential maintained between the condenser plates, as determined by the sparking distance between two needle points was about 28,000 volts, and calculation shows that for an increase of only one elementary charge the increase in $A$ would be small, so that it would be very difficult to measure the amplitudes with a sufficient degree of accuracy to test the ratio $\Delta A/A$.

In view of this fact it was decided to increase the field gradient in a known manner, keeping the charge on the particle constant. This was done by making the distance between the plates at the top different from that at the bottom. A heavy brass strip 3.81 cm. × 1.27 cm. × 8.9 cm. was screwed to the lower edge of the condenser, so that with this in place the two field gradients were in the ratio 2 to 3. The same ratio should, therefore, exist between the amplitudes of the particle's motion at the top and the bottom of the condenser.

Four representative photographs are reproduced. In each of these,

traces of a large number of particles are shown. Many of the drops drift out of focus and in the measurements which follow these were discarded. The data for thirty-six particles is recorded in

TABLE I.

| Drop No. | No. of Waves at Top of Plate. | Dist. in Mm. on Plate. | No. of Waves at Bottom of Plate. | Dist. in Mm. on Plate. | Ave. $\lambda_1$. | Ave. $\lambda_2$. | Amp. at Top in Mm. | Amp. at Bottom. | Ratio of Amplitudes. |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 4 | 32.5 | 5 | 45.3 | 8.12 | 9.06 | 2.4 | 3.6 | .666 |
| 2 | 4 | 37.5 | 4 | 39.5 | 9.37 | 9.87 | 2.6 | 3.5 | .666 |
| 3 | 3 | 34.2 | 4 | 47.2 | 11.40 | 11.80 | 1.6 | 2.6 | .616 |
| 4 | 3 | 33.5 | 4 | 47.7 | 11.16 | 11.92 | 1.6 | 2.0 | .800 |
| 5 | 2 | 39.0 | 2 | 40.0 | 19.50 | 20.00 | .6 | 1.0 | .600 |
| 6 | 3 | 33.0 | 3 | 33.5 | 11.00 | 11.16 | 1.5 | 2.3 | .652 |
| 7 | 3 | 46.0 | 3 | 48.7 | 15.33 | 16.23 | 1.3 | 1.7 | .764 |
| 8 | 4 | 26.7 | 6 | 43.2 | 6.67 | 7.20 | .8 | 1.0 | .800 |
| 9 | 4 | 32.6 | 5 | 41.6 | 8.15 | 8.32 | .5 | .8 | .633 |
| 10 | 6 | 41.6 | 6 | 43.5 | 6.93 | 7.25 | 1.0 | 1.5 | .666 |
| 11 | 3 | 45.5 | 2 | 30.9 | 15.16 | 15.45 | 2.0 | 2.7 | .740 |
| 12 | 5 | 47.2 | 4 | 38.7 | 9.44 | 9.67 | 1.3 | 2.0 | .650 |
| 13 | 3 | 42.7 | 2 | 28.5 | 14.23 | 14.25 | 1.0 | 1.5 | .666 |
| 14 | 11 | 50.0 | 9 | 38.7 | 4.54 | 4.30 | 1.9 | 2.7 | .705 |
| 15 | 4 | 47.2 | 3 | 37.2 | 11.80 | 12.40 | 1.3 | 2.0 | .650 |
| 16 | 3 | 38.2 | 3 | 41.5 | 12.73 | 13.83 | 2.1 | 3.2 | .656 |
| 17 | 5 | 31.2 | 6 | 41.0 | 6.24 | 6.83 | 1.0 | 1.5 | .666 |
| 18 | 6 | 50.5 | 7 | 59.3 | 8.42 | 8.47 | 1.8 | 2.4 | .750 |
| 19 | 5 | 39.5 | 6 | 45.5 | 7.90 | 7.60 | 1.8 | 2.5 | .720 |
| 20 | 6 | 48.5 | 6 | 52.0 | 8.08 | 8.66 | 2.0 | 3.0 | .666 |
| 21 | 4 | 40.0 | 6 | 61.5 | 10.00 | 10.25 | .8 | 1.5 | .537 |
| 22 | 5 | 54.4 | 5 | 55.0 | 10.88 | 11.00 | 1.4 | 2.0 | .700 |
| 23 | 5 | 30.0 | 4 | 24.0 | 6.00 | 6.00 | 2.3 | 3.5 | .656 |
| 24 | 4 | 32.3 | 5 | 40.5 | 8.07 | 8.10 | 1.6 | 2.0 | .800 |
| 25 | 6 | 45.5 | 2 | 13.2 | 7.58 | 7.60 | 1.9 | 2.5 | .760 |
| 26 | 3 | 34.0 | 4 | 45.0 | 11.30 | 11.25 | 1.0 | 1.8 | .555 |
| 27 | 4 | 34.0 | 6 | 52.5 | 8.50 | 8.75 | 1.0 | 1.8 | .555 |
| 28 | 4 | 47.0 | 4 | 47.0 | 11.75 | 11.75 | 2.0 | 2.8 | .714 |
| 29 | 5 | 49.0 | 6 | 59.7 | 9.80 | 9.95 | .6 | 1.0 | .600 |
| 30 | 5 | 56.5 | 5 | 58.3 | 11.30 | 11.66 | 1.3 | 2.0 | .650 |
| 31 | 7 | 49.5 | 6 | 46.5 | 7.07 | 7.75 | 1.0 | 1.5 | .666 |
| 32 | 8 | 61.5 | 8 | 65.5 | 7.71 | 8.19 | .9 | 1.5 | .600 |
| 33 | 8 | 60.0 | 8 | 62.5 | 7.50 | 7.81 | 1.5 | 2.1 | .715 |
| 34 | 5 | 54.5 | 5 | 58.5 | 10.90 | 11.70 | 1.0 | 1.5 | .666 |
| 35 | 8 | 54.0 | 9 | 65.0 | 6.77 | 7.22 | .6 | 1.0 | .600 |
| 36 | 9 | 45.0 | 12 | 60.0 | 5.00 | 5.00 | 1.0 | 1.6 | .625 |

Columns 5 and 6 show the mean wave-lengths for the top and the bottom of the condenser, a wave-length being taken as the distance between successive maximum and minimum in the approximate sine curve reproduced on the plate. The frequency of the 60-cycle alter-

nating current feeding the primary of the transformer was found to be approximately constant at night when most of the work was done, therefore we may compare the wave-lengths of the top and bottom, which should be equal, if the terminal vertical component of velocity is not changed at the lower part of the condenser where the gradient becomes greater.

It will be noted that with three exceptions corresponding to drops Nos. 14, 19 and 26 the wave-lengths are longer in the region where the field gradient is a maximum, and careful observation of the plates showed that in each of these three cases the particles had drifted out of focus so that at the lower end of the fall, the distance measured on the plate was only the projection of the actual motion, which accounts for the apparent discrepancy in the results. We should therefore conclude that a particle falls faster when it is simultaneously vibrating at right angles to the line of fall.

The objection might be raised that the particles under consideration had not reached a constant vertical velocity when they first entered the region of the condenser and so to make sure that this error did not occur photographs were taken of particles falling between parallel condenser plates. The results for six representative particles are shown in the following table and indicate that the terminal vertical velocities had been reached before the particles came under observation.

TABLE II.

| Drop No. | No. of Waves at Top of Plate. | Wave-length in Mm. | No. of Waves at Bottom of Plate. | Wave-length in Mm. |
|---|---|---|---|---|
| 1 | 3 | 11.29 | 3 | 11.29 |
| 2 | 3 | 12.06 | 3 | 12.03 |
| 3 | 4 | 9.20 | 4 | 9.20 |
| 4 | 5 | 8.02 | 5 | 8.01 |
| 5 | 6 | 8.00 | 4 | 8.01 |
| 6 | ·3 | 11.86 | 3 | 11.88 |

Furthermore, the field was somewhat distorted near the middle of the condenser where the half plate terminates. The force lines are curved upward at this point and thus the amplitude of oscillation begins to increase slightly above this region, but this distortion should not give rise to the qualitative increase in fall in the part of the condenser where the field intensity is strongest.

In reference to the amplitudes of the particles, it is interesting to note that a large number of the ratios given in Table I. are approximately two thirds, which would indicate that $k$ is a constant. It is

impossible, however, to draw any definite conclusions from these ratios, for it was observed that the particles usually picked up additional charges near the middle of the condenser, owing to the constant discharge taking place from the sharp edge of the half plate.

If the motions in the two perpendicular directions are independent of each other, we may write the component displacements as

$$x = a \sin \omega t \quad \text{and} \quad y = ct,$$

or the resultant path is given by the equation

$$x = a \sin \frac{\omega y}{c},$$

where $c$ is a constant. The resultant velocity at any instant is therefore

$$v = \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2} = \sqrt{c^2 + a^2\omega^2 \cos^2 \omega t},$$

and makes an angle with the vertical equal to

$$\frac{dx}{dy} = \frac{a\omega}{c} \cos \frac{\omega y}{c}.$$

If we assume a frictional viscous force $F = kv^n$ tangent to the path and resisting the motion of the particle it will have a normal component $F_y$ equal to

$$F \cos \theta = kc(c^2 + a^2\omega^2 \cos^2 \omega t)^{n-1}.$$

If $n = 1$ or if the resistance is proportional to the first power of the velocity, then $F \cos \theta = kc = \text{const.}$ If $n = 1$ then the mean value of $F \cos \theta$ for a half period interval is constant, so that the mean vertical component velocity should be constant within the same interval, and this mean value should diminish as the amplitude increases for the vertical component of the force becomes larger. This would produce a result opposite to the one actually observed.

The maximum horizontal component velocity of the particles was from 10 cm. to 75 cm. per sec., values far in excess of the limit within which Stokes's law holds, so that the resistance in this direction varied as some higher power of the velocity.

If $F_x$ and $F_y$ represent the viscous forces, where each component motion is taken separately, it is evident that they do not represent the component forces when the displacement is a resultant of the separate displacements.

In conclusion it is pointed out that if we assume the resultant viscous force to be exerted tangential to the path at any point, then the terminal vertical component will be decreased rather than increased, so that it is

necessary to assume a turbulent motion of the medium in order to explain this discrepancy from observed results.

If $F_y$ is the resistance offered to a small particle moving with a constant velocity $c$ in the $y$ direction and $F_x$ is the viscous resistance offered to a particle oscillating in the $x$ direction, then it does not follow that $F_x$ and $F_y$ are the components of the viscous tangential resistance when both motions are compounded, unless this resistance depends upon the first power of the velocity.

The experimental results of this paper indicate that if a small sphere, moving through a viscous medium with a terminal velocity $v$ under a constant force is suddenly caused to vibrate at right angles to the original line of motion the vertical velocity component $v$ is increased.

Certain problems[1] relating to the two-dimensional flow of viscous liquids have been solved, but the solution under the boundary conditions set in the present problem involves certain mathematical difficulties. The general equations of motion of an incompressible fluid is of the type

$$\frac{\partial u}{\partial t} = X - \frac{1}{\rho}\frac{\partial p}{\partial x} - v\frac{\partial u}{\partial y} - \omega\frac{\partial u}{\partial z} + \sqrt{\Delta^2 u}$$

where $u$, $v$, $\omega$, $x$, $y$, $z$, are the components of velocity and of body force respectively, $p$ the mean pressure, $\rho$ the density, and $\sqrt{}$ the kinematic coefficient of viscosity. Two general types of solution have been obtained, one in which terms involving the second power of velocity, such as $u(\partial u/\partial x)$ are omitted, and the other in which the terms $\sqrt{\Delta^2 u}$, etc., are neglected. Thus, for steady one dimensional motion, there is a large range of velocities for which no solution has been found to apply. It should be expected, therefore, that the broader problem of motion of a small sphere in two dimensions should present greater difficulties.

A study of the experimental and theoretical problem suggests three possible explanations of the results observed in this paper.

1. "Cavitation" at the edge of the sphere would give rise to the phenomenon observed, since even a small decrease in pressure on the lower side of the drop would give rise to a greater vertical component of velocity.

2. From the experiments of Mr. Ellis Williams[2] the stream lines for large values of $Va/v$ indicate that for a comparatively long distance in the wake of the particle the liquid has the same velocity as the small sphere. If the particle suddenly stops as it does at the end of the oscillation and starts to return the pressure will evidently be greater on the upper side and will increase the vertical fall.

[1] Jeffery, Phil. Mag., 29, 455, 1915.
[2] Phil. Mag., 29, 526, 1915.

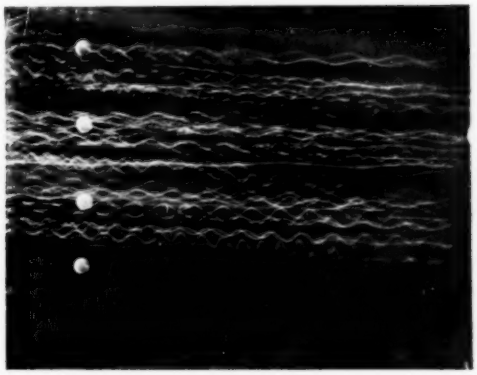


Fig. 1.

Fig. 2.

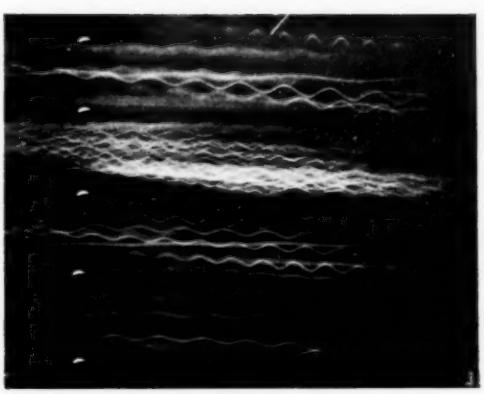


Fig. 3.

W. A. SHEWHART.

3. If the spherical drop of liquid suffers a distortion when it enters the electric field and becomes flattened, it would tend to set itself with the longer axis at right angles to the line of motion. The effect of the field would be to keep the flat side perpendicular to the sides of the condenser. The resultant effect upon the shape of the drop would be complicated, but it would undoubtedly affect the velocity.

In conclusion, I wish to thank Professor E. P. Lewis for his many helpful suggestions and counsel throughout these experiments and for the facilities placed at my disposal.

LABORATORY OF PHYSICS,
UNIVERSITY OF CALIFORNIA,
June, 1916.

# THE USE OF THE EINTHOVEN GALVANOMETER AND A DETECTOR OF THE AUDION TYPE FOR MEASURING THE STRENGTH OF RADIOTELEGRAPHIC SIGNALS.

BY LAURENS E. WHITTEMORE.

ONE of the important determinations to be made in radiotelegraphy is the measurement of signal strength at a receiving station. Measurements of this kind are valuable in determining (1) the effectiveness of various types of antennæ and transmitters, and (2) the effect of weather conditions and of the nature of the region over which propagation takes place.

Among the experiments which have been made in the past to measure quantitatively the phenomena of radio transmission may be mentioned those of Austin,[1] Taylor and Blatterman,[2] and Marchant.[3]

In general, two methods of measurement are in use. (1) Noting the deflection of a galvanometer placed in series with a crystal detector or in shunt around a thermo-junction. (2) Shunting a telephone receiver with a variable resistance which can be adjusted until the signals are just barely audible, that is, until dashes can just be distinguished from dots. The second method has frequently been shown to be unsatisfactory.

For making quantitative measurements it is very convenient to record the signals by means of an Einthoven galvanometer and a suitable photographic device. A permanent record is thereby obtained which can be measured accurately at the leisure of the observer. A very convenient control board for use with this galvanometer is now being manufactured by Leeds and Northrup.[4]

The crystal detectors proving unreliable and the thermo-junctions too insensitive, we may turn to the more recently developed vacuum bulb detector which is sensitive and not easily thrown out of adjustment. A difficulty arises in its use from the fact that a battery of large electromotive force is used in the plate circuit. A current of about a milliampere

[1] Bul. Bur. of Stand., Vol. 7, 1911, Reprint No. 159; Vol. 11, 1914, Reprint No. 226.

[2] Proc. Inst. Radio Eng., 4, 131, 1916. (Good bibliography.)

[3] El. (London), 75, 267, 309, 1915.

[4] H. B. Williams, Am. Journ. Physiol., April 1, 1916.

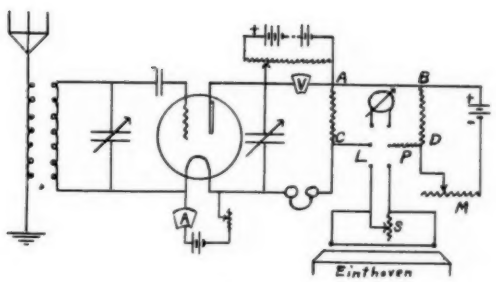



Fig. 1.

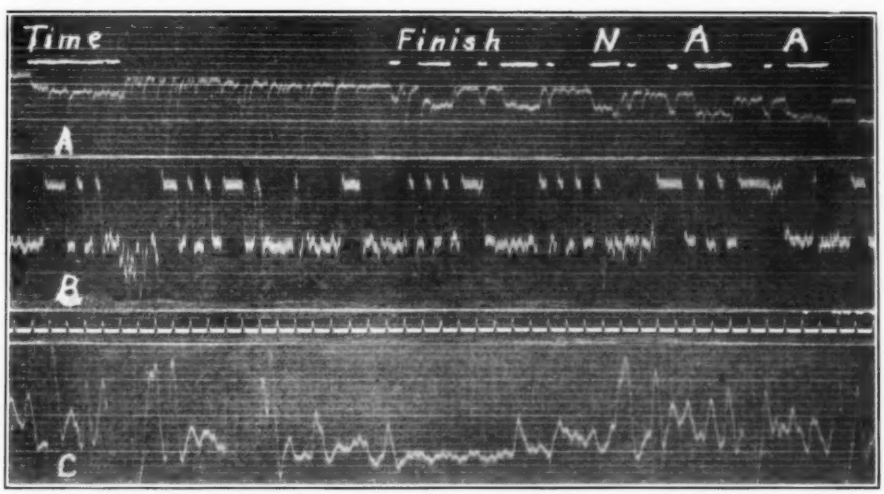



Fig. 2.

LAURENS E. WHITTEMORE.

flows in this circuit whenever the detector is in use, thus making it impossible to use a sufficiently sensitive galvanometer directly in the receiver circuit. To avoid this difficulty the arrangement shown in the accompanying diagram, Fig. 1, was used.

The current which is continually flowing through the plate circuit causes a potential drop to exist across the resistance *A–C* in the circuit. This is balanced by an equal potential drop across a resistance *B–D* in an auxiliary circuit. The two circuits are connected together at the point *A–B* and the galvanometer is placed between the two circuits at *C–D*. When no signals are being received the galvanometer should show no deflection. When signals are received the potential drop *A–C* changes, thereby creating a difference of potential between *C* and *D* and causing a deflection of the galvanometer. If this galvanometer is of the Einthoven type it will follow the variations of the current with the reception of dots and dashes. It was found convenient to keep both of the resistances *A–C* and *B–D* constant at about 2,000 ohms and to secure a balance by varying an additional resistance, *M*, in series with the auxiliary battery.

When the filament of the detector is first lighted it is necessary to wait a few minutes until the current in the plate circuit becomes steady. For the preliminary adjustment of the potentiometer circuit it is very convenient to use a D'Arsonval galvanometer of the box type. It can then be replaced by the Einthoven by means of the switch, *L*, Fig. 1.

For quantitative work it is essential that the detector be put in the same condition of sensitiveness for the successive tests. The detectors of the vacuum bulb type are well arranged in this respect, for it is possible by means of an ammeter in series with the filament and a voltmeter across the battery of the plate circuit to reproduce the conditions in these circuits whenever desired. It is also important to keep the tuning and coupling conditions constant.

A check can be had on the detector by setting up an auxiliary or substitute antenna circuit and exciting this with a constant frequency buzzer in order to give in the detector circuit a current of about the same magnitude as that due to the received signals.[1]

Even after a comparatively steady current has been set up in the receiver circuit (observable with the D'Arsonval galvanometer) a very slow increase or decrease may take place due, perhaps, to fluctuations in the temperature of the bulb, resulting in a drift in the line of photographic signals as in Fig. 2, *A*. In order to bring the galvanometer back to zero

[1] A most excellent arrangement of circuits for this purpose is given by Marchant in the Journal of the (British) Institution of Electrical Engineers, *53*, 329, 1915. Abstract, El., *74*, 621, 1915.

it is necessary to adjust the series resistance, $M$, which consists, conveniently, of a dial type resistance box in connection with a fairly large sliding contact, or otherwise continuously variable, resistance. The deflection caused by the signals is not affected by this adjustment.

The sensitiveness of the apparatus may be decreased for recording strong signals or atmospherics by shunting the galvanometer as at $S$ or including a series resistance, $P$.

Attention may be called to the fact that detector bulbs with lime cathodes have been found rather irregular in their behavior.[1] The newer bulbs of the tubular type seem to be very stable if the filament is not maintained at too high a temperature.

The photographs reproduced in Fig. 2 were obtained by the author by the use of the arrangement described in this paper. In this figure time increases toward the right and the deflection of the galvanometer is down, the zero position being above. $A$ is a photograph of the time dash and signature signals sent out at 10 P.M., eastern time, May 24, 1916, by the U. S. government station (NAA) at Arlington, Virginia, whose distance from this station (9XP) is about 1,000 miles. $B$ shows the beginning of the weather report sent out from Arlington on the evening of April 4, 1916. It reads, USWB — S m(issing). Atmospherics, at times making the signals somewhat difficult to translate, may be seen in $A$ and $B$, while in $C$ (11:15 P.M., March 21, 1916) the atmospherics are so strong as to completely overshadow any signals. The shadow of the time indicator marking fifths of seconds may be seen between $B$ and $C$. The parallel lines, one millimeter apart, are for convenience in measurement and are caused by rulings on the cylindrical lens in front of the shutter of the camera.

This method of measurement seems especially applicable for taking data such as is being gathered by the British Association for the Advancement of Science, through its committee for radiotelegraphic investigation, for obtaining quantitative information regarding the influence of the weather on atmospherics and signal strength.

I am indebted to Dr. T. Townsend Smith, following whose suggestion the arrangement above described was developed, and to Dr. Ida Hyde, of the department of physiology of the University of Kansas, who was so kind as to put the Einthoven galvanometer at my disposal.

BLAKE PHYSICAL LABORATORY,
UNIVERSITY OF KANSAS.

[1] Willows, El., *74*, 742, 1915.

# PROCEEDINGS

OF THE

# AMERICAN PHYSICAL SOCIETY.

## A Simple Method for Determining the Audibility Current of a Telephone Receiver.[1]

By Edward W. Washburn.

THE use of the telephone receiver as a detector for small alternating currents in wireless telegraphy, in many electrical measurements with the alternating current bridge, and for other purposes makes it desirable to have a simple and reliable method for determining the audibility current of a telephone. By audibility current is meant the current which must pass through the telephone in order to produce a sound which can just be recognized. The curve connecting the audibility current of a telephone with the frequency of the impressed E.M.F. is a valuable indication of the field of usefulness of the instrument and its availability for the above purposes.

The usual method of determining such a curve is first to measure the volt sensitivity of the telephone at various frequencies. This is usually done with the aid of a suitable slide wire connected in shunt across a non-inductive branch of an alternating current circuit provided with a hot wire ammeter.[2] Having measured the volt sensitivity in this way, the audibility current is then obtained by dividing the audibility voltage at each frequency by the impedance of the telephone for that frequency. Since, however, both the effective resistance and the inductive reactance of the telephone depend upon the frequency, it is necessary in this method to measure each of these quantities for each frequency. Moreover, as pointed out by Austin,[3] it is doubtful whether the values thus obtained are strictly applicable for the purpose in hand, since they are measured with the aid of currents very much larger than the audibility current of the telephone. This uncertainty can be avoided and the whole determination much simplified by means of the following method based upon the equation of the Wheatstone bridge.

The set-up is shown in the accompanying figure. The resistances $R$ and $R'$ should be substantially free from both inductance and capacity. For low

[1] Abstract of a paper presented at the Chicago meeting of the American Physical Society, Dec. 2, 1916.

[2] Cf. Wien. Ann. Phys., 4, 456 (1901).

[3] N. B. S. Bull., 5, 155 (1908).

resistance telephones Curtis coils would fulfill this condition sufficiently well, but for high resistance telephones especially at the higher frequencies, film resistances[1] would be preferable. The two resistances marked $R'$ are connected by a stretched wire of suitable resistance ($R_w$), provided with a scale and a sliding contact. The telephone is connected to this sliding contact and to a point between the two resistances $R$ as indicated. A small variable air condenser connected where it is required is employed to insure the exact balancing of reactances. The resistance of that portion of the slide-wire, lying between its central point and the position occupied by the sliding contact

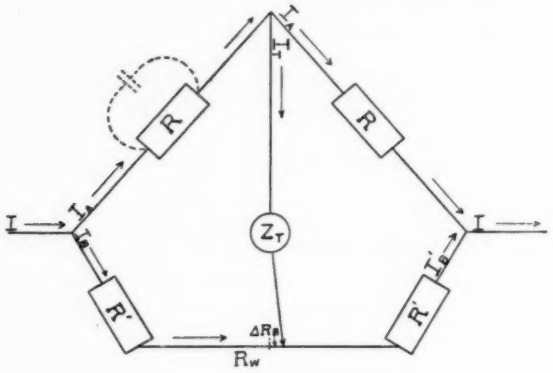



Fig. 1.

when the audibility current $I_T$ is passing through the telephone, will be designated by $\Delta R_B$.

The current $I$ comes from a high frequency generator whose frequency can be controlled and kept constant at any desired value. It enters the bridge network at the left and divides as indicated. In series with the bridge is placed a vacuum thermocouple (not shown in the figure) connected to a sensitive millivoltmeter. This instrument indicates the magnitude of the current $I$. For high resistance telephones a hot wire ammeter could be employed.

The procedure consists in adjusting the current $I$ until the range of silence on the slide-wire is found to have a convenient measurable value. This range of silence is obviously equal to $2\Delta R_B$ ohms. The value of $I_T$ is then obtained from the equation of the Wheatstone bridge which is as follows:[2]

$$I_T = \frac{2\Delta R_B R I}{A(A + 2\dot{Z}_T)},$$

where $A$ is written in place of $R' + \frac{1}{2}R_W + R$.

This method would seem to involve a knowledge of $\dot{Z}_T$, $(= R_T - jx_T)$ the impedance of the telephone, but this can obviously be avoided by making $A$ so large that $2\dot{Z}_T$ is negligible in comparison with it, or, so large that the D.-C. resistance of the telephone may, with sufficient accuracy, be employed in place of its impedance. For low resistance telephones there will be no

[1] Jour. Amer. Chem. Soc., *35*, 179 (1913).

[2] Maxwell, Electricity and Magnetism, I., 477.

difficulty in doing this because the impedance of a low resistance telephone is ordinarily not more than two or three times its resistance even at fairly high frequencies. In the case of some of the very high resistance wireless telephones, where the impedance at 1,000 cycles may run up as high as 16,000 ohms it will, of course, be necessary to determine and employ an approximate value of its impedance. For an electrically tuned telephone $Z_T$ in the above equation reduces to $R_T$, the effective resistance.

As an example of the determination of the audibility current of a telephone receiver by this method, the following values may be cited:

The high frequency generator was operated at a frequency of 990 cycles per second. Resistance boxes of Curtis coils were employed for obtaining the two resistances indicated by $R$, each box being set at 5,000 ohms. The value of $R' + \frac{1}{2}R_w$ was 125 ohms. The range of silence on the slide-wire was found to be $2\Delta R_B = 0.025$ ohm. The vacuum thermocouple indicated a current $I = 0.4$ milliampere. The D.-C. resistance of the telephone was 170 ohms and its effective resistance at 1,000 cycles was 220 ohms, its impedance at this frequency being 290 ohms. For $A$ we have, therefore, 5,125 ohms. The value of $2Z_T$ could evidently be neglected in comparison with 5,125 ohms; or, the value of $2R_T$, that is, 340 ohms could be used in place of it, since in either case the error involved would be less than ten per cent. Substituting these values in the above equation, we find that the audibility current of this telephone is $2.10^{-6}$ milliampere at 1,000 cycles.

Department of Ceramic Engineering,
University of Illinois,
November 15, 1916.

## NEW BOOKS.

*First Course in General Science.* By F. D. BARBER, Professor of Physics, Illinois State Normal, M. L. FULLER, Forecaster, U. S. Weather Bureau, J. L. PRICER, Professor of Biology, Illinois State Normal, and H. W. ADAMS, Professor of Chemistry, Illinois State Normal. New York: Henry Holt and Co., 1916. Pp. vii + 607.

This book is another addition to the books which have been planned for use as texts in a first course in science, to be given in the first year of a high-school course. Wisely there is no attempt in this book to cover the entire field of elementary science; instead those things with which the pupil is already more or less familiar have been selected. The authors believe that a first course of this kind should deal almost wholly with physical sciences. The only biological part is a chapter on microörganisms. On the whole the subject matter seems to be well selected.

O. M. S.

*General Physics.* By W. S. FRANKLIN AND B. MACNUTT. New York: McGraw-Hill Publishing Co., 1916. Pp. viii + 604. Price $2.75.

The authors state in the preface that this book has been prepared to meet the needs for a more rigorous treatment of the subject, such as used in the teaching of mathematics. Further, the notation of the differential calculus is used to some extent. This may be somewhat misleading, for the text is really intended as a first course for students that have not had calculus. Mechanics and some parts of heat may be more difficult than in some of the other recent texts. In part this is due to the fact that some of the simpler things, familiar topics already understood by the student, are omitted. Simple machines, the pulley, inclined plane, etc., are not mentioned. Some of the more difficult topics in electricity have been left out. No discussion of the magnetization of iron, with a distinction between the intensity of a magnetic field, $H$, and magnetic induction, $B$, is given. Permeability is not mentioned. The term potential does not appear, instead of difference of potential, as for example, between the plates of a condenser, electromotive-force is used.

The treatment of heat is very interesting. It is approached from the standpoint of thermodynamics with considerable emphasis on the law of entropy.

There is among engineers considerable confusion in the use of the word "*pound.*" Is it a unit of force, or a unit of mass, or is it to be used for both mass and force? Textbook writers on physics, probably wish that engineers would answer this question, and possibly engineers wish that the physics

textbook writers would settle it. The authors of this book use *pound* as a unit of mass, and as unit of force, the *poundal*, and for work, the *foot-poundal*. The result, of course, is that an engineer using this book for reference and wanting an answer in foot-pounds may have trouble, for example, in computing the kinetic energy of either translation or rotation. But why *poundal*, and *foot-poundal*? Are they used any place outside of textbooks? Certainly engineers do use forces a great deal, but on the other hand they seldom use the concept of mass. Hence the common practice among them is to use the pound as the unit of force.

However no brief review of this book can do justice to it. There are here many ingenious methods of handling difficult topics, and it abounds with helpful analogies.

O. M. S.

*Engineering Applications of Higher Mathematics.* By V. KARAPETOFF. New York: Wiley and Sons, 1916. Part II., Problems on Hydraulics. Pp. v + 101. Part III., Problems on Thermodynamics. Pp. v + 113. Part IV., Problems on Mechanics of Materials. Pp. v + 81. Part V., Problems on Electrical Engineering. Pp. vii + 65. Price, $.75 each.

The problems contained in these small volumes appear to be very well chosen to fulfill the purpose of the author as stated in the prefaces: to assist "the student or the engineer who wishes to review calculus or analytics or to acquire facility in applications of higher mathematics to engineering problems." They are "practical" in a legitimate sense of that much misused word. The engineering student who goes through one or more of these books, even though he be of very ordinary mathematical ability, can hardly escape the conviction that the calculus is a real and powerful tool of his trade and not an educational frill devised mainly for his discomfort.

The problems of different types are introduced by brief and clear statements of the physical and mathematical principles involved and this part of the work is, on the whole, admirably done. Sometimes, especially in the volume on thermodynamics, the brevity of the explanation leads to inadequate or even erroneous definitions and statements of the facts of nature which underly the formulæ and methods used. A student would gain a very imperfect comprehension of absolute temperature or entropy from this volume, but it is to be remembered that it is not designed to be a textbook but a sort of laboratory manual in applied mathematics and that it should be accompanied by a book which is much fuller on the theoretical side.

The problems are of about the same grade of mathematical difficulty as those ordinarily found in textbooks of mechanics which assume a knowledge of the elementary calculus; but the author very sensibly introduces certain valuable methods not usually contained in such books. Thus in Chapter III. of Part III. and in Chapter VI. of Part IV. much practice is provided in dealing with empirical equations, determining their constants and improving their

form; the method of least squares is utilized without much theoretical introduction but with insistence upon what is usually the only practicable way of using the method, viz., by applying it to find small corrections to approximate values of the constants.

Apart from their regular use with a class, these little books are a valuable storehouse of good problems which ought to be of service not only to teachers of engineering and of mathematical physics but perhaps even more to teachers of analytical geometry and the calculus. There can be little doubt that the average college student misses the real point in a good deal of his mathematical study; a great many men get their first adequate and rational idea of the calculus when they come to apply it to mechanics or physics, as every teacher of these subjects knows. There is much to be said in favor of a greater admixture of "practical" problems in the mathematical courses themselves and these books will be helpful to teachers who desire to do this.

H. A. B.

*The Emission of Electricity from Hot Bodies.* By O. W. RICHARDSON. New York: Longmans, Green and Co., 1916. Pp. vi + 304. Price, $2.75.

After a brief history of the subject, general statements concerning the theory of ions and ionization, the conductivity of gases drawn from the neighborhood of hot bodies, the ratio of the charge to the mass of an ion and the electron theory of metallic conduction, and brief descriptions of experimental methods of obtaining gas-free bulbs and of measuring temperatures, the author discusses the theory of the emission of electrons from hot bodies. This he does from the point of view of thermodynamics, the kinetic theory of gases and the quantum hypothesis, developing two principal formulas, which later he puts to experimental tests. This part of the book also contains sections on contact differences of potential, the liberation of electrons by chemical action, photoelectric emission by the complete spectrum (black body radiation), etc.

Later chapters are devoted to the effect of gases on the emission of electrons from platinum, tungsten and other substances, the energy transformations connected with the emission of electrons, the emission of positive ions from hot metals and the influence of gases on it, the emission of ions by heated salts and ionization due to chemical reactions, bubbling, splashing, etc.

The book does not contain descriptions of the important practical applications of the subject in wireless telegraphy and telephony, X-radiation, long-distance telephony, etc.

To a large extent (although not entirely) the book represents researches carried on by the author and his co-workers. A scientific investigater has taken the time to write out a connected account of his own researches and related matter, and for this reason, as well as on account of the importance and interest attached to this new subdivision of science, the book should find a place in every library that claims to have an approximately complete set of works on physical science.

*Problems in Physics.* By WILLIAM D. HENDERSON. New York: McGraw-Hill Book Company, Inc., 1916. Pp. viii + 205. Price, $1.50.

The book contains 1,025 problems in mechanics, sound, heat, electricity and light, designed for engineering students who are taking a first course in university physics, but who have not yet reached the calculus stage.

The solutions of most of the problems involve the simple substitution of numerical values for the algebraic symbols in the equations that represent some of the fundamental and at the same time elementary laws of physics. These equations are to be found in brief statements of fundamental scientific principles, that precede the various subdivisions into which the book is divided.

A few examples are solved in the text. Answers are not, in general, given.

The book will doubtless be of considerable value to those (both teachers and students) who are engaged in first-year engineering physics.

W. D.

*The Physical Properties of Collodial Solutions.* By E. F. BURTON. New York: Longmans, Green and Co., 1916. Pp. vii + 200. Price $1.80.

The subject of collodial solutions is one which has become most attractive by virtue of its extensive applications. In this connection it is necessary to mention only such fields of investigation as dyeing, the properties of clay, the manufacturing of soaps, etc. The subject, however, has been almost repellent owing to its complication and more specially to the desire of different investigators to introduce so many different words. The first investigations were of course those of Graham, about 1861. For a long time there were comparatively few additions to the knowledge obtained from his experiments. Then came an enormous revival of interest and at the present time the literature is practically overwhelming. As a matter of fact a distinctly new science has arisen, with its textbooks, its special journals and special societies.

This book by Dr. Burton, of the University of Toronto, is one of the Monographs on Physics, edited by Sir J. J. Thomson and Dr. Horton. In every respect it is admirable. It states clearly the various terms used by different writers and with great success brings order out of chaos in the analysis of different types of solutions. The references to the literature are wonderfully ample. The statement of theories is in every case clear and the conclusions drawn are as definite as one could expect under the circumstances. The two indexes are most helpful. In short, no book has yet appeared in English or any other language which brings so many facts together, so well coördinated and so well described. It is indispensable to any one interested in the modern developments of this branch of physical chemistry.

The scope of the book and the width of its point of view are best shown in a few words by giving the list of the titles for the different chapters. They are as follows: Preparation and Classification of Collodial Solutions; The Ultra-Microscope; The Brownian Movement; The Optical Properties of Collodial Solutions; Measurements of the Sizes of Ultra-Microscopic Particles; Motion

of Collodial Particles in an Electric Field; The Coagulation of Colloids; Theory of the Stability of Colloids; Practical Applications of the Study of Colloidal Solutions. It will be apparent from this that the author has brought to bear on the subject his extensive knowledge of physics, mathematics and chemistry; and the combination has resulted in a most satisfactory treatise.

J. S. A.